Comparison of Arching Profiles of Golden Age Cremonese Violins and Some Mathematically Generated Curves

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Abstract— Three mathematically generate curves are compared to the arching profiles of the plates of six golden age Cremonese violin family instruments. Generated curves include the circular arc, curate cycloid and sinusoidal curve. Images are presented for visual comparison and a simple measure of fit is calculated for each arching profile and generated curve combination. A subjective rating of the fit of generated curves to profiles is also performed. Results indicate that the circular arc shows the closest average fit for the longitudinal arching, but the fit is only moderately close. The curtate cycloid curve shows the closest average fit for the transverse arching profiles, with very good fit shown in 27% of comparisons. Variability among the profiles sampled is too high to reliably indicate that any one of the generated curves serves as a reasonable general model for these arching profiles. The sinusoidal curve consistently under represents the transverse profiles and so is not a likely candidate as a model for an average transverse profile. The curtate cycloid curve also consistently under represents the transverse profiles at the upper and lower bout locations and so is also not a likely candidate as a model for an average profile at these locations.

I. Introduction

A cursory examination of historical violin construction texts yields little information on the derivation of arching profiles of the plates of that instrument. And a small sampling of modern violin makers indicated that many if not most modern builders derive arching profiles in a very ad hoc fashion, beginning carving with some idea of edge thickness and overall height but carving mostly by eye, the process possibly aided by the use of arching templates made from plans of classic instruments or the builder's own previous work. Articles written by Quentin Playfair [1, 2] suggested the possibility that violin family plate arching may be described by mathematically generated curves and, by extension, that historical builders may have used such curves as models for arching profiles. In those articles a single type of mathematically generated curve, the curtate cycloid, was positioned near arching profiles from classic instruments for visual comparison. Close fits appear in a number of cases. Playfair hypothesizes that the curtate cycloid curve may have been used as the model for plate arching by Cremonese violin makers but concludes that not enough historical evidence is available to test this hypothesis. However, that work is suggestive enough to warrant a more detailed investigation of the subject.

This paper outlines a further investigation of the possibility that mathematically generated curves describe arching profiles of classic violins. It includes a uniform comparison of three generated curve types to some of the arching profiles of six instruments. This investigation used as its working hypothesis that arching profiles of existing Cremonese violins may be modeled by one of three mathematically generated curves – the circular arc, curtate cycloid and sinusoidal curve. As compared to Playfair's hypothesis of use of one of these curves by golden age Cremonese violin makers, this hypothesis is testable by simple comparison of instrument arching profiles to generated curves.

II. Experiment

A. Sample Population

Profiles of the five violins and one viola shown in **table 1** were studied. The instruments were chosen at random. All arching profiles were taken from drawings on the back of posters of instruments in The Strad poster series. It is unknown how representative this sample is of the population of extant golden age Cremonese instruments, but the instruments selected were chosen with no previous knowledge of their arching profiles. These six instruments were simply chosen as the first six posters available from the Strad library poster website

[1] Manuscript received February 20, 2011. R.M. Mottola, Liutaio Mottola Stringed Instrument Design, http://LiutaioMottola.com, rmm at LiutaioMottola dot com.

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(http://www.orpheusmusicshop.com/category-31.html) at the time. Transverse profiles and a limited number of longitudinal profiles were studied.

The total number of arching profiles studied was a subset of the theoretical number of profiles that should be available. Ideally, instrument drawings would include longitudinal profiles for both top and back plates, and transverse profiles for all three bouts and at both corner positions. But the poster drawings varied greatly in the number of profiles provided. Not included in this study were any profiles at the corner positions, as the recurve length of the profile was quite long and quite variable in these areas, and this adversely affected the ability to accurately define the nadir of the recurve, which would have affected the analysis. Also excluded from analysis were any profiles which were not complete or were drawn in two parts, as using these would require reconstruction on my part, which could result in error. Finally, back longitudinal profiles that did not include information that would make possible the exclusion of the button area from the analysis (another potential source of error) were also excluded. No profiles from these instruments were excluded from the study for any other reason.

Ref#	Description
1	Nicola Amati 'Alard'1649
2	Andrea Guarneri 'Conte Vitale' 1676 (viola)
3	Joseph Guarneri Filius Andrea c. 1705
4	Antonio Stradivari 'Viotti' 1709
5	Antonio Stradivari 'Kruse' 1721
6	Guarneri Del Gesu 'Kreisler' 1733

Table 1 – The arching profiles of these golden age Cremonese violins were examined in this study. All profiles were taken from drawings from The Strad posters available from The Strad Library, and are used with permission.

B. Mathematically Derived Curves

The study compared each half profile (from nadir of recurve to center line, for the transverse profiles) to each of three mathematically generated curves – the circular arc, the curtate (contracted) cycloid and a sinusoidal curve. These curves were chosen for a number of reasons. The circular arc serves as a good reference curve as it is simple to generate and readily visualized. It also looks like a potentially good model for longitudinal arching profiles. The curtate cycloid has potential as a model of the transverse profiles, as described in the aforementioned papers by Playfair. The sinusoid was included as it also appears similar to some transverse arching profiles. Each of these curves was included because, in addition to mathematical generation, they can be generated by simple mechanical means. Although not the primary focus of this study, the possibility exists that one or more mathematical curves could have served as models for golden age violin makers. Although the math describing all of these curves was not available at the time these instruments were built, the means to approximate all of these mechanically were readily available. Formulae and mechanical methods for each curve are described briefly below. Note that each curve can be completely described by two parameters, length and height. Values for these are readily obtainable from the instrument arching profiles. For purposes of comparison the arching profiles were maintained as arrays of Cartesian coordinates (x,y pairs) with a fixed x interval. The formulae for the generated curves provide coordinates for any point on the curve. A complete description of the data input and processing procedures appears in a later section.

The y coordinate of a point ***x,y*** on a **circular arc** of length ***l*** and height ***h*** can be described by equations (1) and (2):

$$r = \frac{h^2 + l^2}{2h} \quad (1)$$

$$y = h + \sqrt{r^2 - (l - x)^2} - r \quad (2)$$

where ***r*** is the radius of the arc. A circular arc can be generated mechanically with a compass and can be approximated by mechanical means for shallow curves by a simple bent spline.

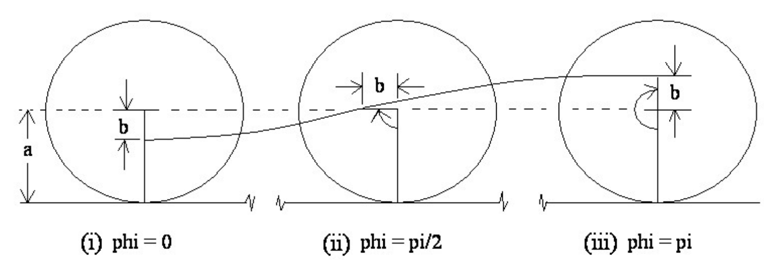

Figure 1 – A half curtate cycloid curve can be generated mechanically by a rolling disk with phase angle beginning at 0 and ending at pi. (Diagram by Dave Cohen from his *American Lutherie* article "Curtate Cycloid Arching", used with permission)

A point ***x,y*** on a **half curtate cycloid** of length ***l*** and height ***h*** can be described by parametric equations (3) and (4):

$$x = a\varphi - b\sin\varphi \quad (3)$$

$$y = a - b\cos\varphi \quad (4)$$

where ***a*** is the radius of the circle the circumference of which is equal to 2***l***, ***b*** = 0.5***h*** and ***φ*** is the phase angle in radians. A half curtate cycloid curve can be generated mechanically by rolling a wheel of circumference 2***l*** with a pencil placed in a hole positioned at a distance ***b*** from its center point through ½ revolution starting with the pencil directly below the center point as shown in **figure 1**. More detail on the math and the mechanical generation of a cycloid for arching profile purposes can be found in Dave Cohen's *American Lutherie* article "Curtate Cycloid Arching" [3], and in the articles by Playfair. Interesting features of half curtate cycloid curves include:

- given a fixed length, the basic shape of the curve is a function of its height;
- as the height decreases relative to the length the curve becomes more sinusoidal in shape;
- as height increases from near zero to the maximum height possible relative to length, the point of inflection moves from a point near half the length and half the height of the curve to a point near the initial point of the curve.

The y coordinate of a point ***x,y*** on a **sinusoidal curve** of length ***l*** and height ***h*** as used here to model half of an arching profile can be described by equations (5)-(7):

$$A = \frac{h}{2} \qquad (5)$$

$$\omega = \frac{\pi}{l/2} \qquad (6)$$

$$y = A * |\sin(\omega x + 1.5\pi)| + A \qquad (7)$$

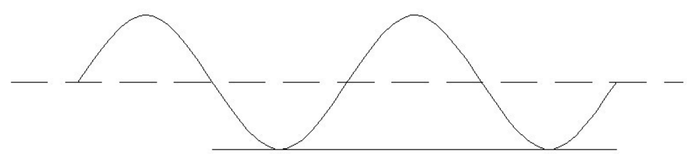

Figure 2 – A full length sinusoidal curve as used in this study can be visualized as a segment of a sine wave starting at a valley and ending at the next valley. A half curve starts at a valley and ends at the next peak.

This curve can be visualized as a segment of a sine wave beginning at a "valley" and ending on the following peak as shown in **figure 2**. The full length curve can be approximated by mechanical means for small heights by clamping the ends of a bent spline so they are collinear, as shown in **photo 1**. Interesting features of half cycle sinusoidal curves are that, given a fixed length, the basic shape of the curve is independent of the height, and the point of inflection is located at x=l/2,y=h/2.

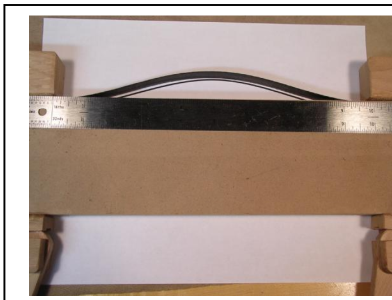

Photo 1 – A (full) sinusoidal curve can be mechanically approximated by a bent spline with ends clamped so as to be collinear. Here a thin strip of plastic is clamped to a board at both ends. The drawing underneath is of a computer generated sinusoidal curve.

C. Design of the Study

The first major issue in the design of this study was establishing a uniform procedure for comparing the generated curves and the arching profiles of the instruments. Both the size of the generated profiles relative to the instrument profiles, and their positioning, also relative to the instrument profiles, must be established in a uniform manner if study results are to be generalized to an instrument population larger than just those instruments examined. The length and height of each instrument profile were easily determined. As half profiles were studied, it made sense to fix the high points (centerline points) of each instrument profile and the generated curves to which it was compared at the same point. But deciding where to fix the plate edge end of the generated curves was more problematic. Criteria considered important in the selection of this terminal point included:

- It should be located at or near the plate edge so the length of the profile for comparison is as long as possible;
- It should be simple to unambiguously locate the point;
- The point should terminate a curve that is of the same general class as the mathematically generated curves.

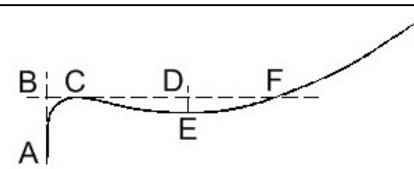

Figure 3 – Locations considered to anchor the edge end of generated curves for purposes of comparison. Point E is at the nadir of the recurve, point F is the nominal end of the recurve.

Various candidate locations were considered as indicated in **figure 3**. All meet the first criterion above. Those points related to the recurve meet the second criterion as well, except in the case of profiles at the corner positions of the violin plates. The wide, vague and generally asymmetrical curves here tend to make it difficult to unambiguously identify the nadir point. But the nadir of recurve point ideally met the last criterion, at least for the cycloid and sinusoid curves. The half arching profile of the violin can be classified as a curve that has terminal points with zero slope and a point of inflection somewhere along its length. This description also describes both the half cycle cycloid and sinusoid curves, and so it is at least possible to achieve perfect fits using these generated curve types. Note that the circular arc does not fit this description, but some preliminary analysis indicated that this was not a likely candidate to model the transverse arching profiles anyway.

For this study point **E**, the nadir of the recurve, was chosen as the terminal point for the generated curves. Given the problems of unambiguously locating this point for the transverse arching profiles at the corner positions, these profiles were not considered in this study.

The second major issue in the design of the study was to consider how to provide a quantifiable measure of fit between an arching profile and a generated curve. A simple measure of average distortion per unit length was chosen. This is described in more detail in a subsequent section. This proved to be adequate for the purposes of this study. A simple subjective visual scoring of fit was also performed as a way to qualify the range of data from the numerical analysis. This was a simple close enough/not close enough evaluation, and it is also described in more detail later.

D. Data Input, Processing and Display

All instrument arching profiles to be included in the study were scanned into bitmap files at 600 dpi using a calibrated flatbed scanner. The bitmap images were digitized into a series of x,y coordinates using **WinDig** digitizing software written by Mr. Dominique Lovy of the Dept. of Physical Chemistry at the University of Geneva and available for free download at http://www.unige.ch/sciences/chifi/cpb/windig.html. The software is designed to digitize printed graphs, and generates a y pixel value for each x pixel value in a graph line. This means the output array of coordinates has a fixed x coordinate interval of 1 pixel. At 600 dpi, this translates to an x interval of ~0.042mm. It should be noted that the thickness of the lines representing the profiles of the drawings varied and were as much as ~0.42mm thick. The scanning software is designed to hew to the center of a drawn line, but with no indication of the methods and resolutions used to transcribe and print these lines on the posters this value should be taken as an error factor when considering subsequent data comparisons.

The coordinate array representation of each arching profile was then input into an Excel spreadsheet for further processing and analysis. Array values were offset to origin coordinates 0,0 and then rotated as necessary to compensate for any misalignment between the baseline of the profile and the scanner bed. Coordinate values were then converted from pixels to millimeters. Each full profile was divided in half so each half profile could be processed separately. The right half of the profile was mirrored so that both halves could be viewed in the same orientation. The nadir of the recurve was located for each half profile as the lowest point in the recurve area.

Since each half arching profile is rendered as a series of coordinates with a constant x interval, generating the mathematical curves as a series of coordinates with the same x values facilitates analysis and display. The formulae given earlier for y values of points on the circular arc and sinusoidal curves were used in the spreadsheet to generate coordinates for these curves. The parametric equations for the curtate cycloid curve cannot be used in this manner as they yield both x and y values as a function of phase angle. A separate script was used to generate y values as a function of x values by successive approximation for the curtate cycloid curve (available at http://LiutaioMottola.com/formulae/curtate.htm). These values were input into the spreadsheet for further analysis.

The last processing step was to compare each half arching profile to each of the three generated curves. Because each curve is realized as an array of coordinates with the same x values and a fixed x interval, it is a simple matter to calculate the area between each arching profile and each generated curve. This area is used as the basis for a simple single measure of fit, average distortion per unit length, specified in units of mm^2/mm. Tables of the set of these values and of summary statistics are provided below.

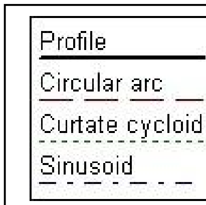

Figure 4 – Legend of line types and colors used in the following figures.

It is important in a study like this to be able to see what the comparisons look like. Individual images containing one half of each instrument arching profile compared to all three generated curves are included in the following figures (**5 – 17**). Note that these are grouped by location on the instrument, i.e. all back lower bout soundpost side profiles are grouped together, etc. This presentation is advantageous as similarities (and differences) among the sampled instruments are more apparent than if all profiles from a single instrument are grouped together. To make visual comparisons easier, profiles for the Conte Vitale viola (instrument #2) are normalized to typical violin profile lengths. All display images were generated with **Graph 4.3**, an open source mathematical graphing software package written by Ivan Johansen and available for free download at http://www.padowan.dk/graph/.

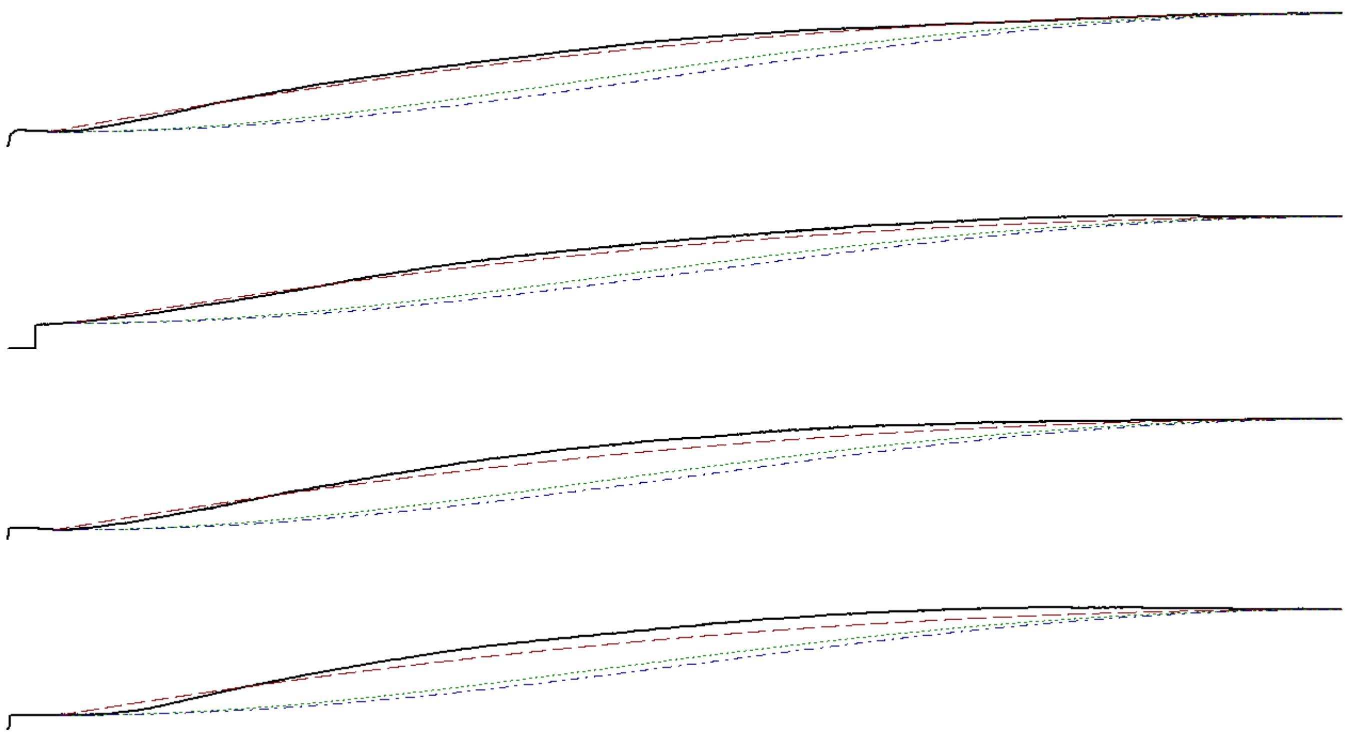

Figure 5 – (top to bottom) back neck end, back tail end, top neck end, top tail end profiles for instrument #3, the only instrument for which reliably scan-able longitudinal profiles were available from the plans.

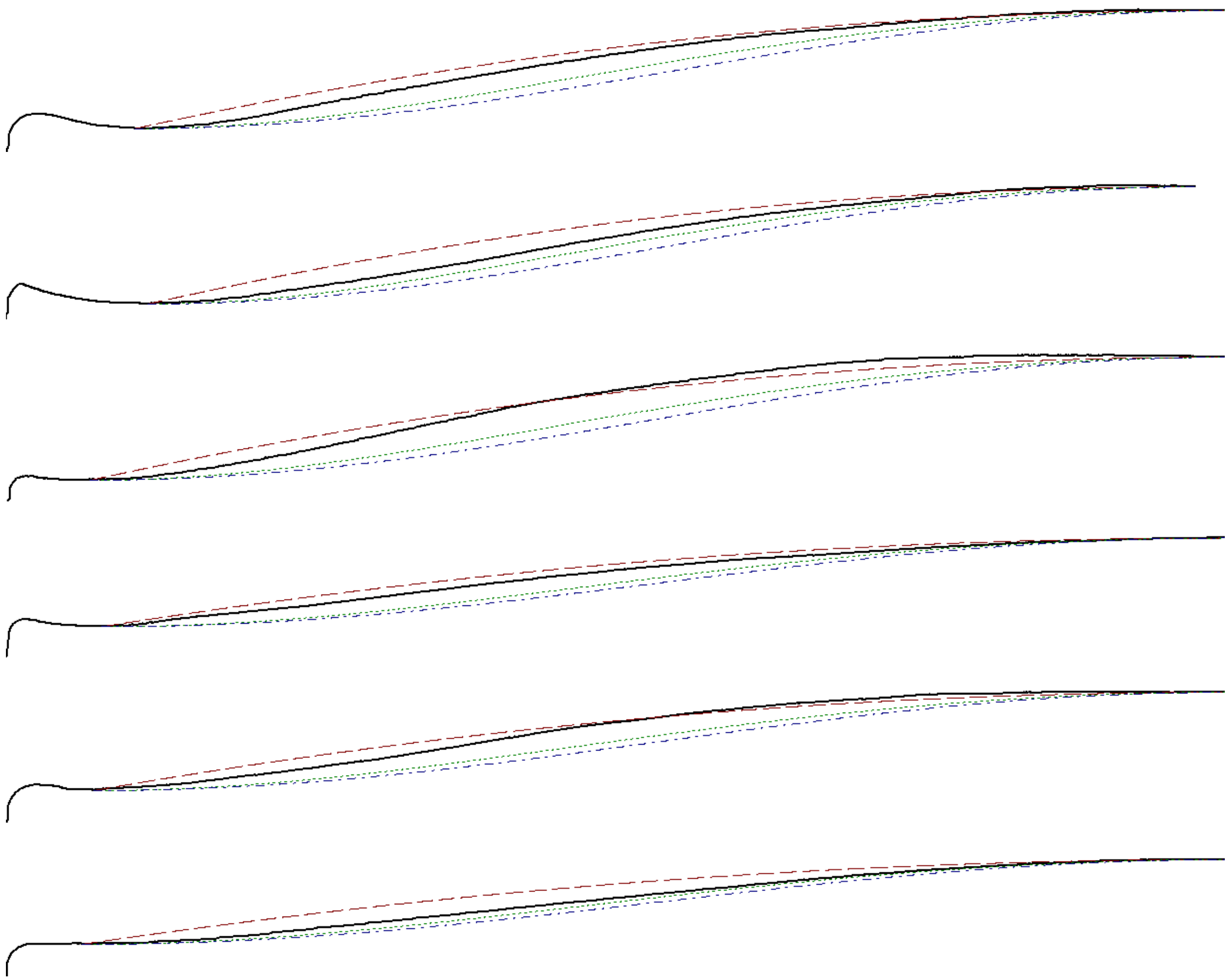

Figure 6 – Back lower bout soundpost side transverse arching profiles for instruments #1-6.

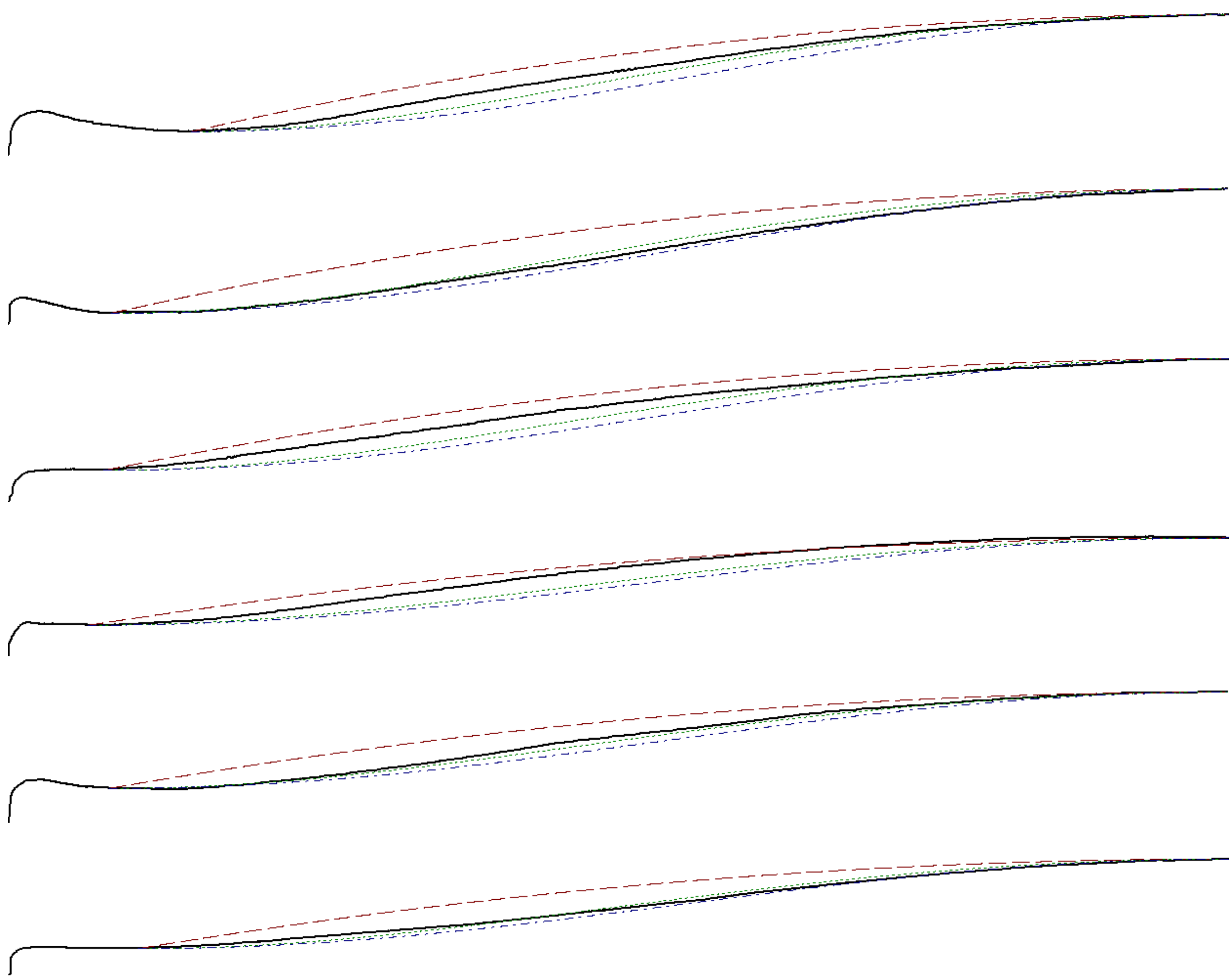

Figure 7 – Back lower bout bassbar side transverse arching profiles for instruments #1-6.

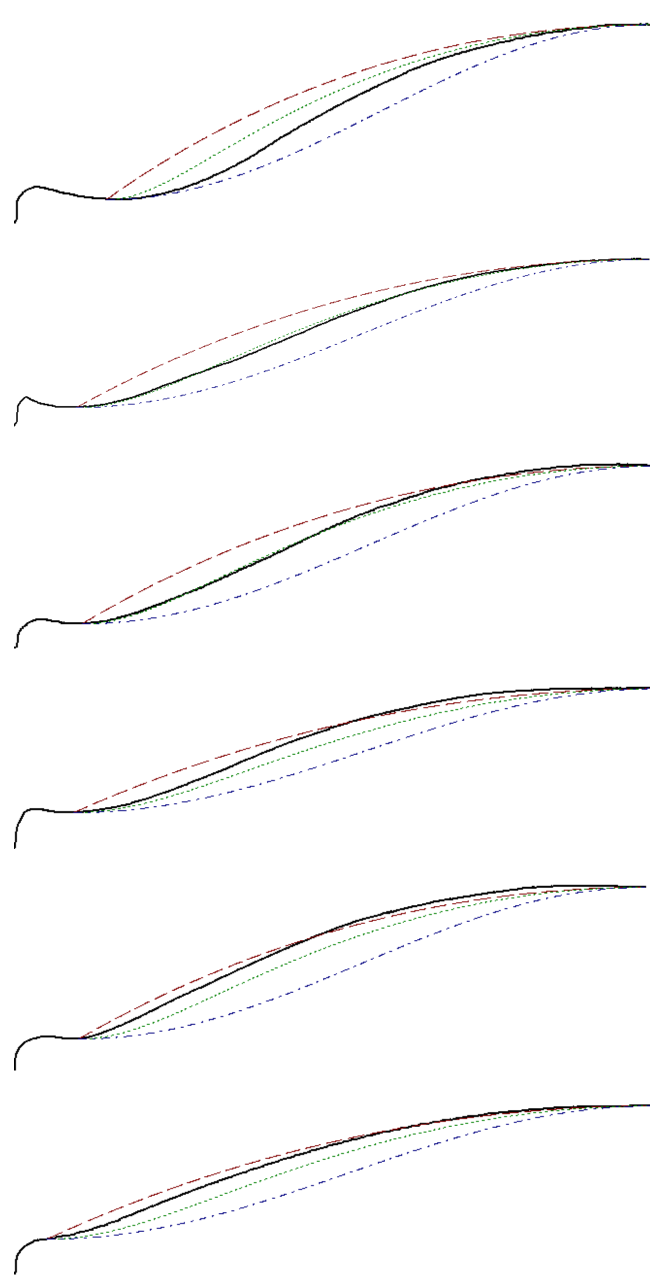

Figure 8 – Back C bout soundpost side transverse arching profiles for instruments #1-6.

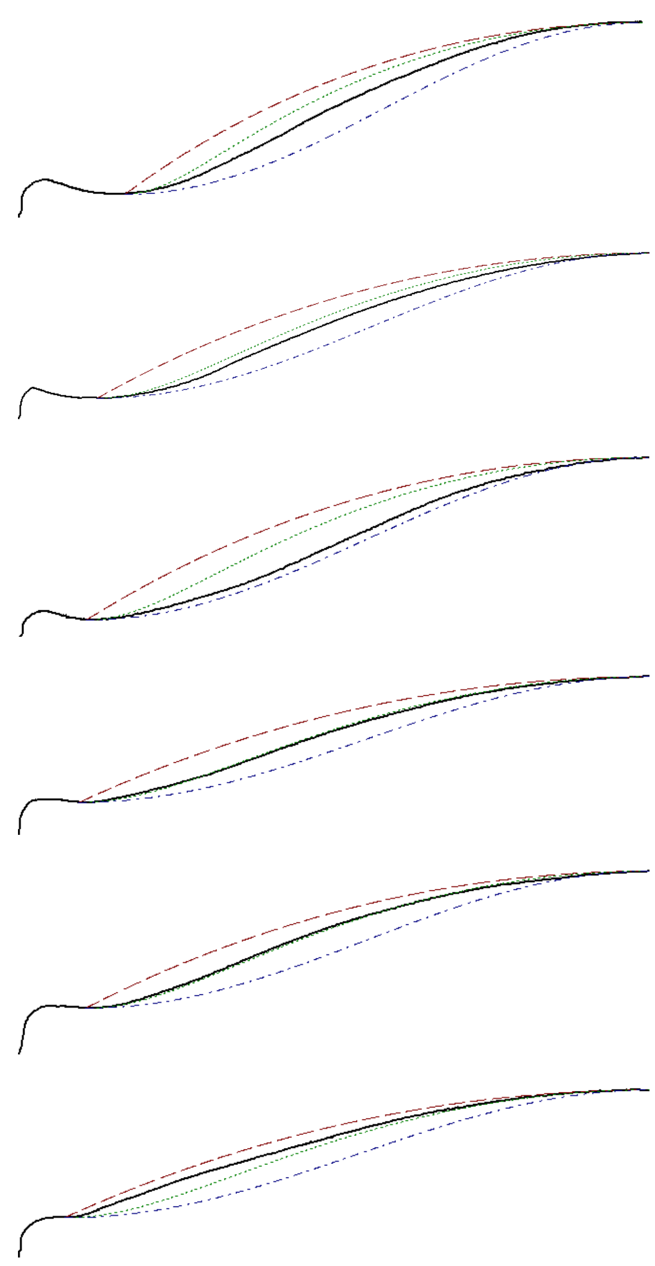

Figure 9 – Back C bout bassbar side transverse arching profiles for instruments #1-6.

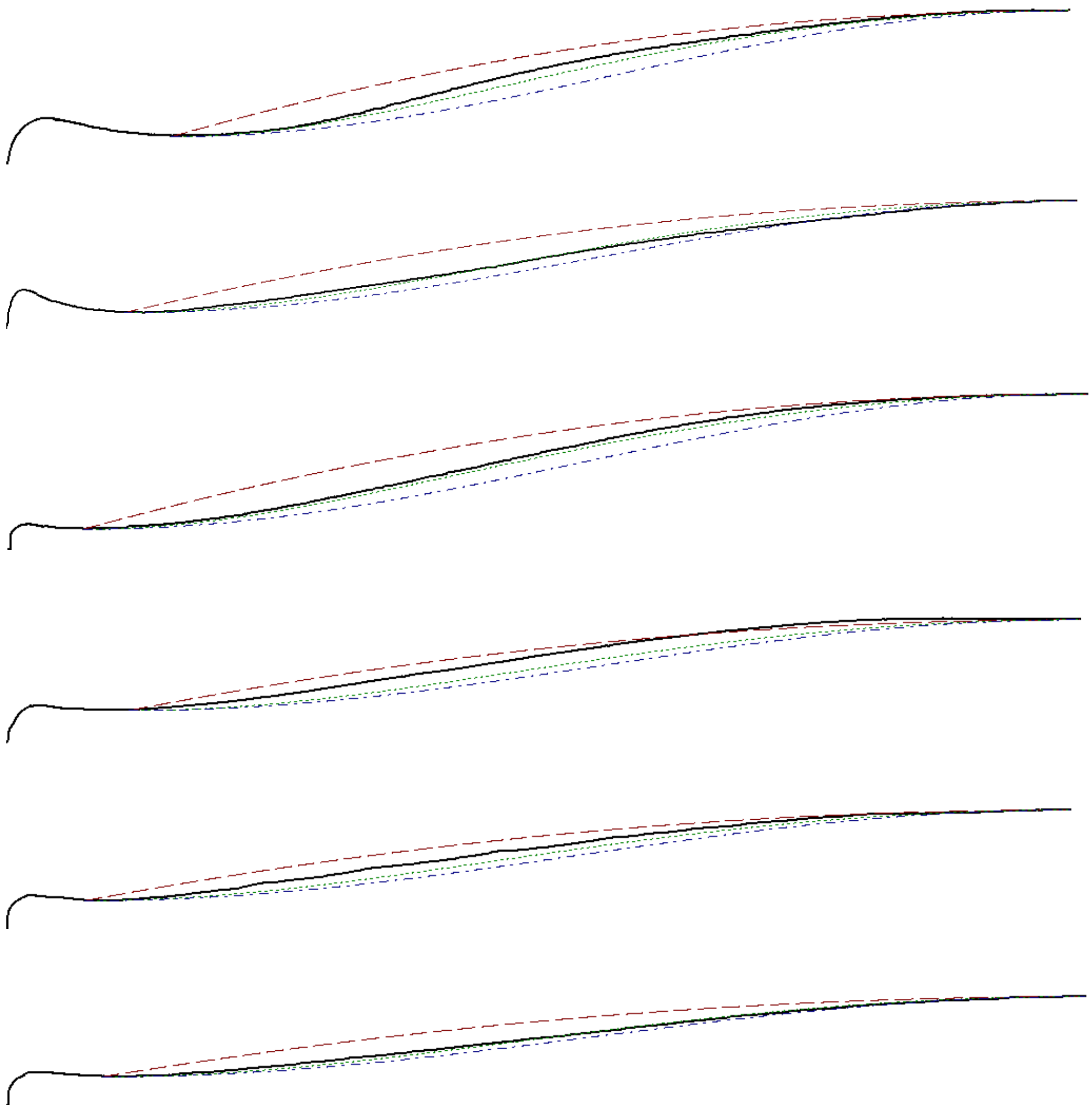

Figure 10 – Back upper bout soundpost side transverse arching profiles for instruments #1-6.

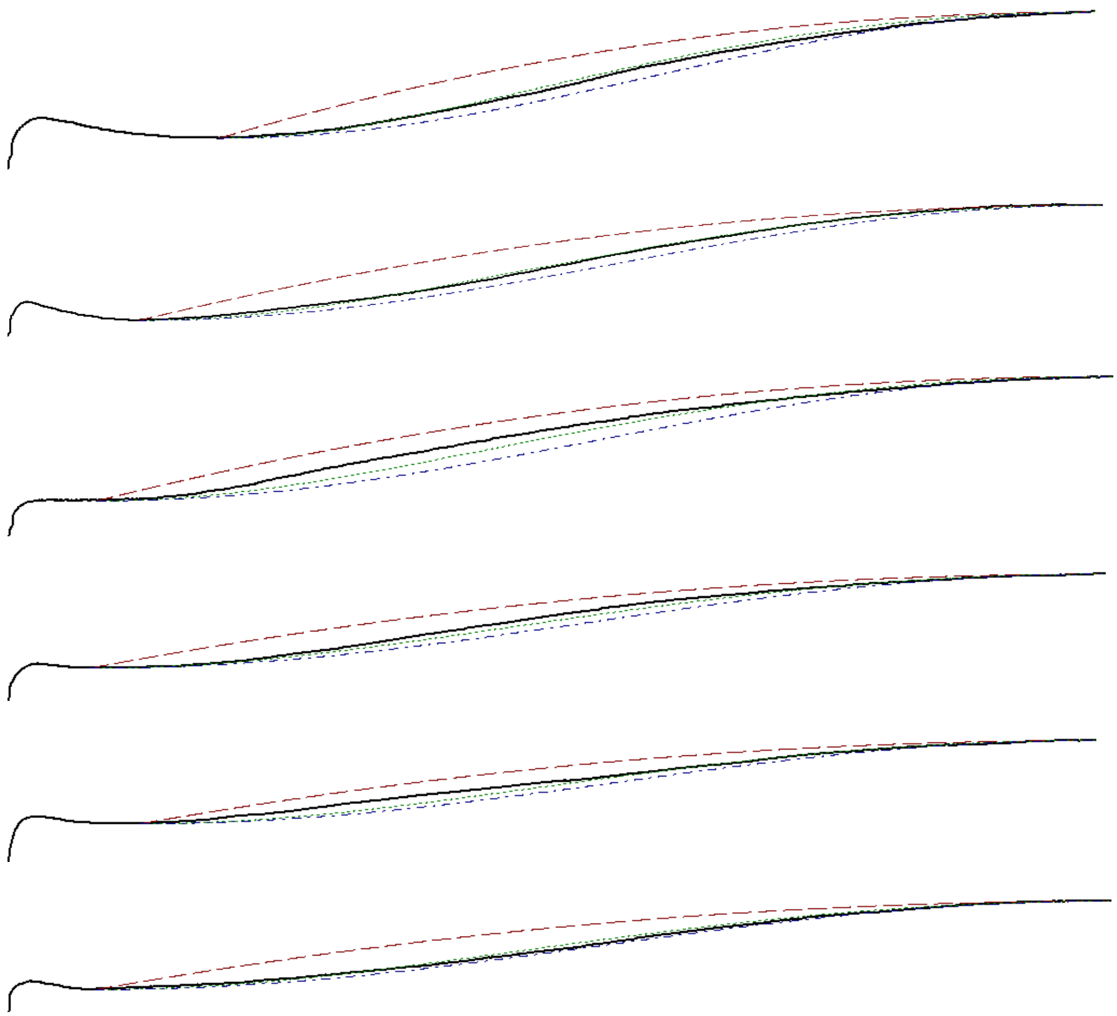

Figure 11 – Back upper bout bassbar side transverse arching profiles for instruments #1-6.

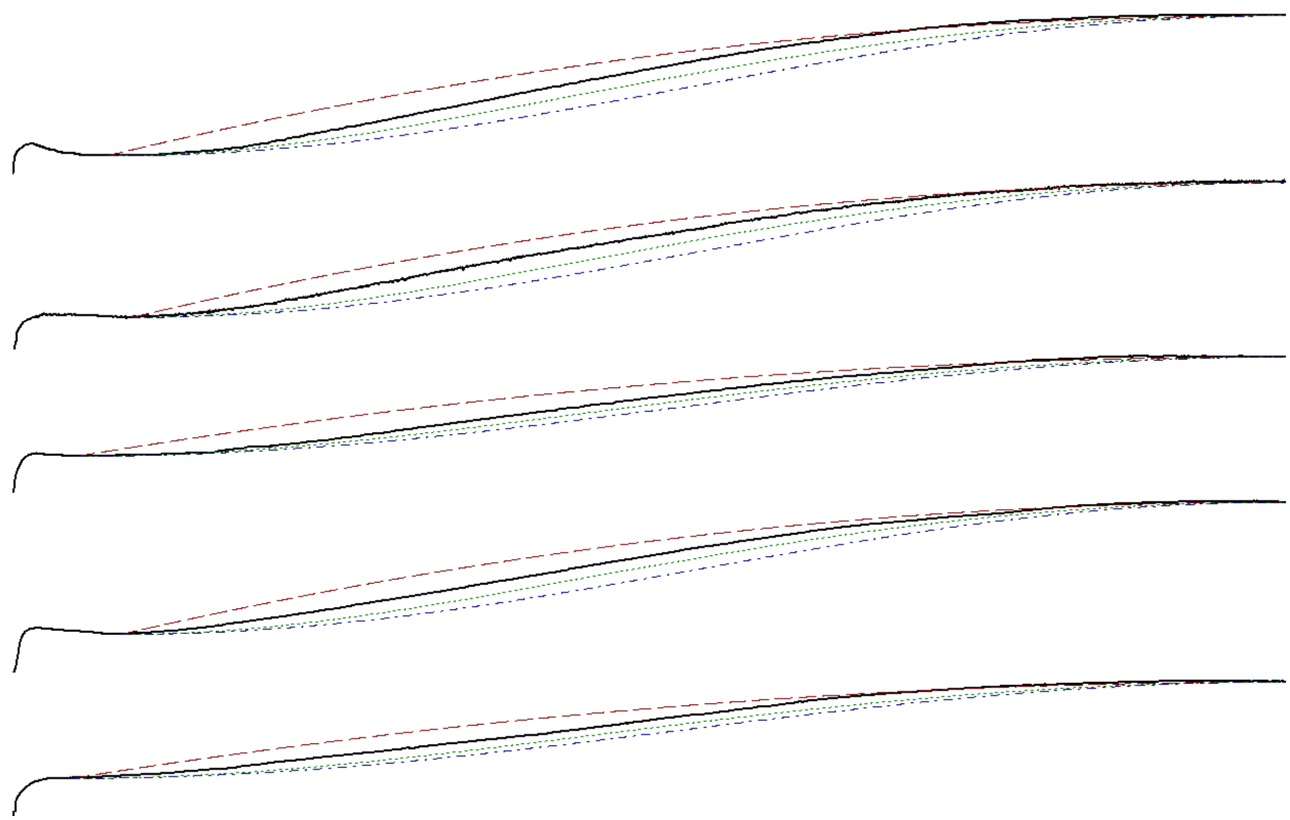

Figure 12 – Top lower bout soundpost side transverse arching profiles for instruments #2-6 (this profile not available for instrument #1).

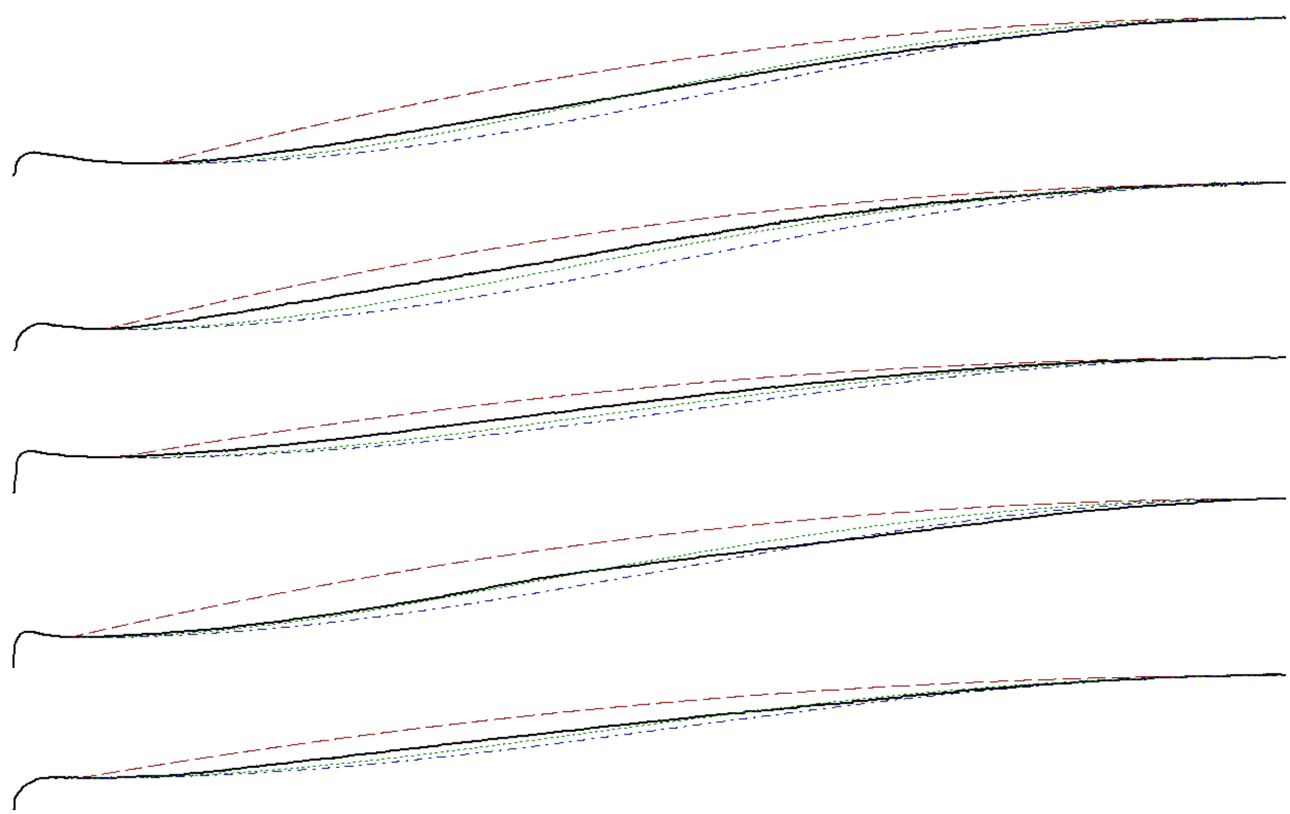

Figure 13 – Top lower bout bassbar side transverse arching profiles for instruments #2-6 (this profile not available for instrument #1).

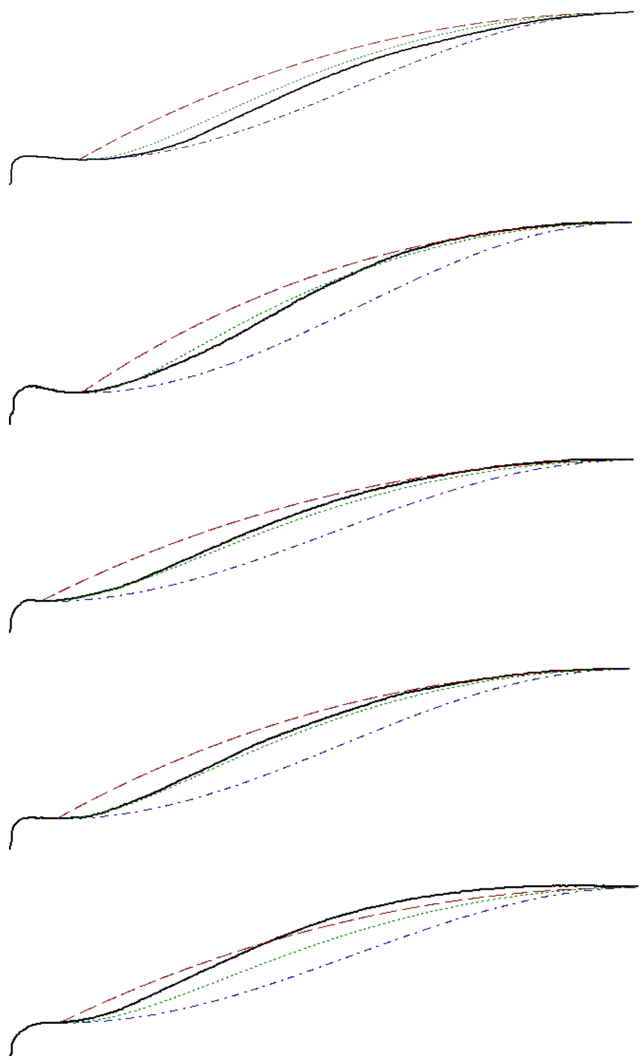

Figure 14 – Top C bout soundpost side transverse arching profiles for instruments #2-6 (this profile not available for instrument #1).

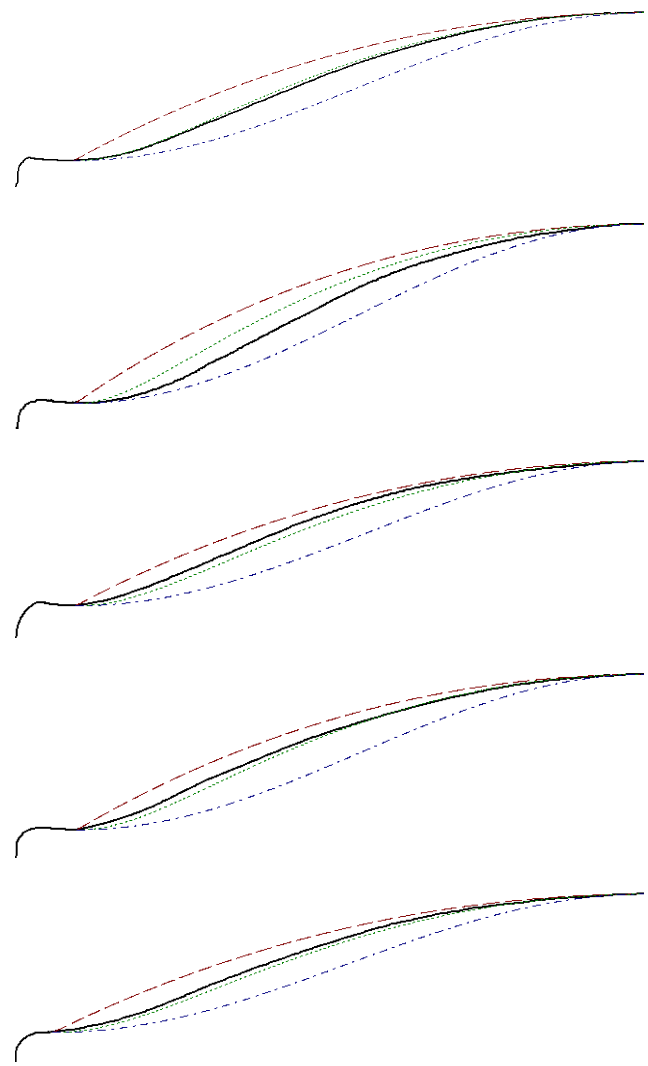

Figure 15 – Top C bout bassbar side transverse arching profiles for instruments #2-6 (this profile not available for instrument #1).

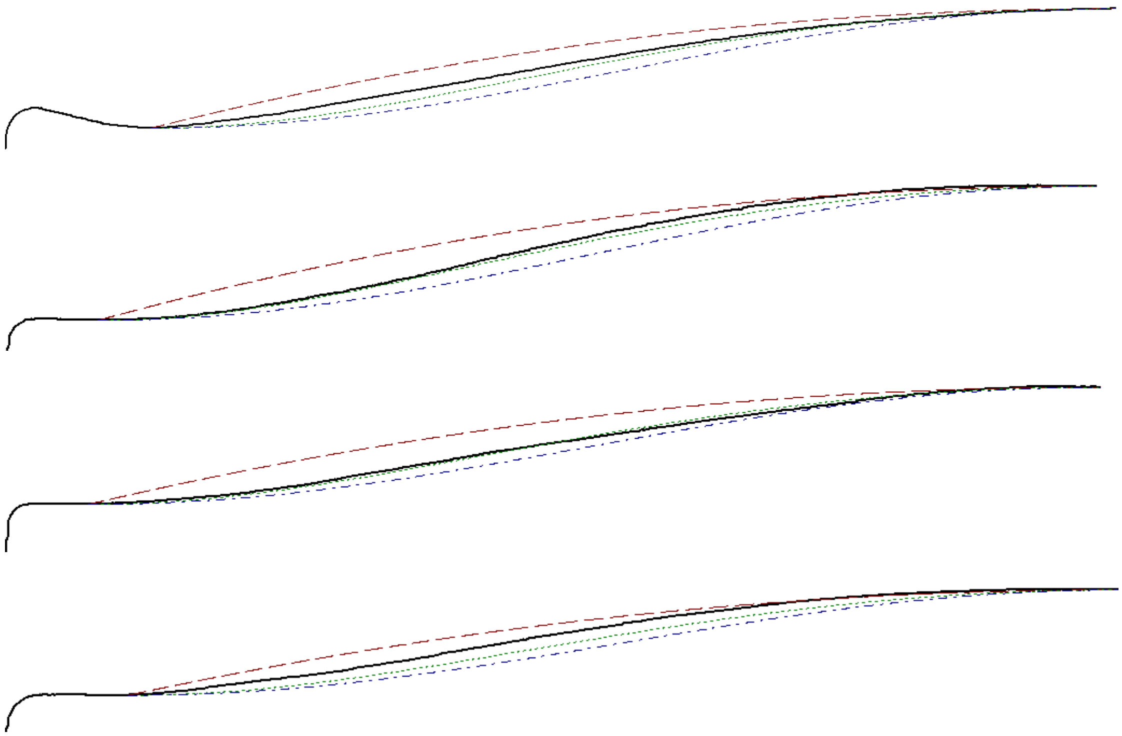

Figure 16 – Top upper bout soundpost side transverse arching profiles for instruments #2,3,4,6 (this profile not available for instrument #1 or #5).

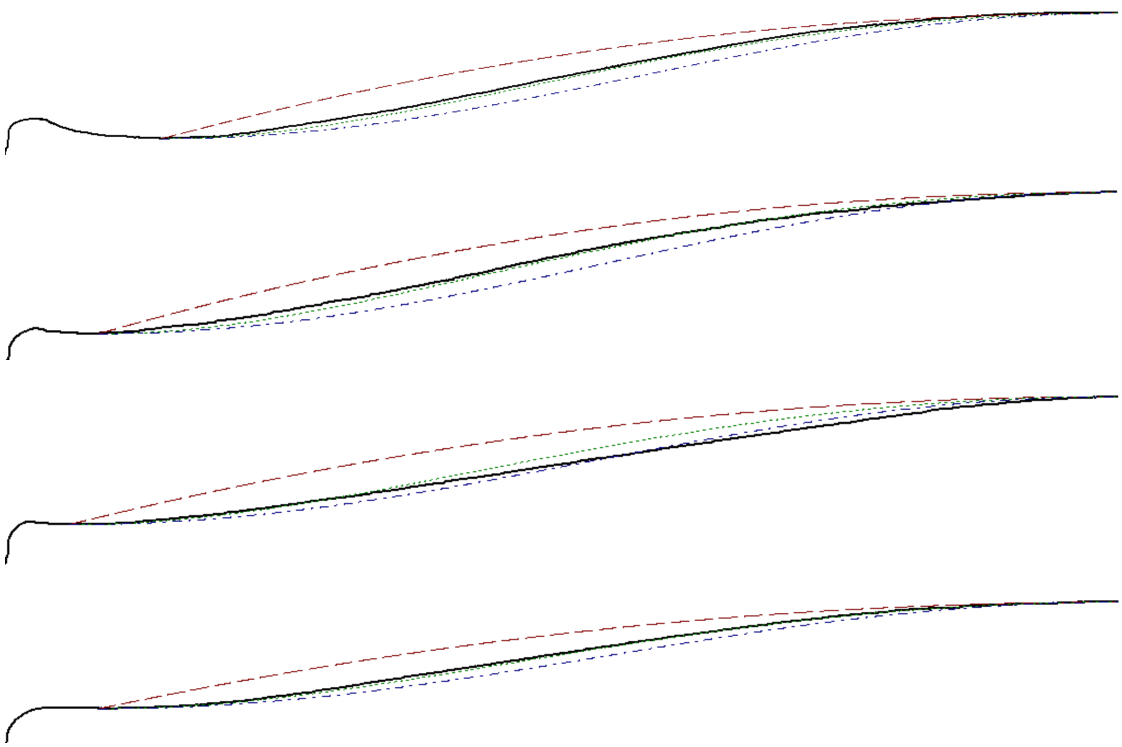

Figure 17 – Top upper bout bassbar side transverse arching profiles for instruments #2,3,4,6 (this profile not available for instrument #1 or #5).

E. Analysis

As mentioned the data were analyzed using a single measure of fit, average distortion per unit length of the profile, presented in units of mm^2/mm. Thus the smaller this value the better the fit. These average distortion per unit length values were calculated for all three generated curve types compared to the longitudinal profiles, and for both cycloid and sinusoidal curves compared to the transverse profiles. Some summary statistics based on this data were also calculated.

A subjective visual scoring of fit was also performed before doing the above analysis, comparing the cycloid and sinusoid curves to the transverse profiles. This scoring was performed before the numerical analysis was complete, to keep the numerical results from biasing the subjective scoring. For each profile/curve combination a simple binary ranking was made. A value of 1 was assigned to a combination that was assessed to represent a fit that was “close enough”, and a value of 0 was assigned to each other combination. Obviously these results are quite subjective and scores would vary from assessor to assessor. But comparing this scoring to the results of the numerical analysis provided some indication of what the average distortion per unit length values might mean in practical terms to a builder. Those interested in this study are encouraged to perform this subjective scoring themselves.

Table 2 contains average distortion per unit length values for all longitudinal profiles studied and for all three generated curve types. Unfortunately longitudinal profiles were only available for a single instrument. **Table 3** contains both average distortion and subjective score values for all transverse profiles, but for only the curtate cycloid and sinusoidal curves. **Table 4** contains summary data for all transverse profiles. Because the summary data in **table 4** show such high variability, summary statistics were also calculated for various subsets of the transverse profiles. Summary data by location of profile on the instrument, by side of the instrument, by plate, and by instrument are presented in **tables 5 – 8**.

Inst.#	Plate	End	Circular Dist.	Cycloid Dist.	Sinusoid Dist.
3	back	neck	0.30	2.18	2.72
3	back	tail	0.49	2.22	2.67
3	top	neck	0.63	2.36	2.83
3	top	tail	0.86	2.51	2.94

Table 2 – Distortion per unit length values for the longitudinal profiles studied.

III. Discussion

A number of general trends are apparent from just looking at the figures. Great variability in shape of profiles taken from the same location on the instrument is apparent. It is also readily apparent that the two sides of the same profile are often noticeably different from each other.

The circular arc appears to approximate the longitudinal profiles moderately well for the one instrument for which those profiles are available, except at the body ends. Much of this distortion at the end could be attributed to the fact that the circular arc curve does not have zero slope at this end point. Neither the cycloid nor the sinusoidal curve approximates these profiles well.

For the transverse profiles, the circular arc never describes the actual profile well. For this reason this combination was eliminated from the data tables for these profiles. Great variability in the fit data for both the cycloid and sinusoidal curves is readily apparent from both the range and standard deviation values in the summary statistics. Range, standard deviation and average values for the sinusoidal curve were about twice those for the curtate cycloid curve. This, plus the fact that only one profile was judged acceptably close to the sinusoidal curve in the subjective evaluation eliminates the likelihood that the sinusoid is a suitable means of modeling these arching profiles. For this reason, summary data for the sinusoidal curve is not shown in **tables 6 – 8**. Note that in each comparison of the sinusoidal curve to a transverse profile the sinusoidal curve is generally under the profile over most of its length. Poor fit of this curve to the profiles is therefore unlikely to be attributable just to the great variability in shape of the profiles.

The curtate cycloid curve showed lower range, average and standard deviation values than the other model curves and also had the highest score in the subjective evaluation (17 out of 62 or 27%). For this reason the discussion below will focus just on this curve as a model for the transverse profiles. Note that in each comparison of the curtate cycloid curve to a transverse profile at both the upper bout and lower bout locations, the curtate cycloid curve is generally under the profile for most of its length. At the C bout location though, the curtate cycloid curve is in some cases above and in some cases below the profile for most of its length. The former may indicate that even though this curve showed the best fit of the curve types analyzed, it may still be suboptimal and there may be another curve type that shows a better fit.

Inst.#	Plate	Bout	Side	Cycloid Dist.	Sinusoid Dist.	Cycloid Subjective	Sinusoid Subjective
1	back	lower	soundpost	0.75	1.15	0	0
1	back	lower	bassbar	0.36	0.74	0	0
1	back	upper	soundpost	0.25	0.76	0	0
1	back	upper	bassbar	0.20	0.36	1	0
1	back	C	soundpost	0.75	1.04	0	0
1	back	C	bassbar	0.69	1.14	0	0
2	back	lower	soundpost	0.54	1.04	0	0
2	back	lower	bassbar	0.35	0.24	0	0
2	back	upper	soundpost	0.22	0.41	0	0
2	back	upper	bassbar	0.11	0.44	1	0
2	back	C	soundpost	0.15	1.49	1	0
2	back	C	bassbar	0.55	1.02	0	0
2	top	lower	soundpost	0.70	1.31	0	0
2	top	lower	bassbar	0.35	0.67	0	0
2	top	upper	soundpost	0.35	0.82	0	0
2	top	upper	bassbar	0.18	0.73	1	0
2	top	C	soundpost	0.65	0.94	0	0
2	top	C	bassbar	0.17	1.36	1	0
3	back	lower	soundpost	1.19	1.60	0	0
3	back	lower	bassbar	0.51	0.80	0	0
3	back	upper	soundpost	0.20	0.72	1	0
3	back	upper	bassbar	0.34	0.70	0	0
3	back	C	soundpost	0.19	1.49	1	0
3	back	C	bassbar	1.11	0.40	0	0
3	top	lower	soundpost	0.57	1.08	0	0
3	top	lower	bassbar	0.40	0.96	0	0
3	top	upper	soundpost	0.21	0.72	0	0
3	top	upper	bassbar	0.18	0.64	1	0
3	top	C	soundpost	0.26	1.57	0	0
3	top	C	bassbar	0.74	1.04	0	0
4	back	lower	soundpost	0.58	0.79	0	0
4	back	lower	bassbar	0.64	0.85	0	0
4	back	upper	soundpost	0.62	0.88	0	0
4	back	upper	bassbar	0.16	0.39	1	0
4	back	C	soundpost	0.64	1.50	0	0
4	back	C	bassbar	0.12	0.82	1	0
4	top	lower	soundpost	0.31	0.56	0	0
4	top	lower	bassbar	0.34	0.60	0	0
4	top	upper	soundpost	0.13	0.38	1	0
4	top	upper	bassbar	0.45	0.28	0	0
4	top	C	soundpost	0.27	1.39	0	0
4	top	C	bassbar	0.34	1.48	0	0
5	back	lower	soundpost	0.86	1.13	0	0
5	back	lower	bassbar	0.14	0.40	1	0
5	back	upper	soundpost	0.33	0.57	0	0
5	back	upper	bassbar	0.21	0.38	0	0
5	back	C	soundpost	0.84	2.14	0	0
5	back	C	bassbar	0.11	1.07	1	0
5	top	lower	soundpost	0.54	1.00	0	0
5	top	lower	bassbar	0.33	0.43	0	0
5	top	C	soundpost	0.22	1.49	0	0
5	top	C	bassbar	0.22	1.53	0	0
6	back	lower	soundpost	0.21	0.40	0	0
6	back	lower	bassbar	0.20	0.21	1	0
6	back	upper	soundpost	0.11	0.22	1	0
6	back	upper	bassbar	0.14	0.12	1	1
6	back	C	soundpost	0.65	1.58	0	0
6	back	C	bassbar	0.41	1.29	0	0
6	top	lower	soundpost	0.45	0.69	0	0
6	top	lower	bassbar	0.20	0.37	0	0
6	top	upper	soundpost	0.50	0.82	0	0
6	top	upper	bassbar	0.06	0.37	1	0
6	top	C	soundpost	0.98	2.01	0	0
6	top	C	bassbar	0.23	1.26	0	0

Table 3 - Distortion per unit length values and subjective scoring for the transverse profiles studied.

The great variability among the instruments and between the two sides of the same profile is an issue, whether the goal is to identify a simple curve to acceptably describe the arching profiles of these instruments as they exist (the focus of this study), or to ruminate on the possibility of simple curves being used by the original makers as models. Considering for a moment the latter case, there would be three major potential causes of deviation from a model curve in real instruments. The first and most obvious is simply that the different builders used different model curves. The second is build variation [4], the differences between instances of a model of instrument that result from variability in manufacturing processes, including the possibility that the instruments were not built to any definite model at all. Build variation is impossible to assess with this sample population, and is likely impossible to assess given access to all extant golden age Cremonese instruments. One clue to the extent to which build variation exists can be gleaned from the differences in height of the edge lip on the two sides of the same profile of most of the instruments. Unless damage and/or repairs have affected this area (very likely unfortunately, as this is an area prone to wear and other damage), these height differences are more likely to be the result of manufacturing process variation than of changes to the shape of the instrument due to use and age. And this is the third potential major cause of variability among the instruments – shape changes due to stresses imposed on the structure due to string tension and other factors over time. It is possible to speculate to some extent on this – for example, the soundpost side of some of the back waist profiles is higher in the soundpost area than it is on the other side of the instrument.

Cycloid Dist.	Sinusoid Dist.	Cycloid Subjective	Sinusoid Subjective
Averages All		Total	
0.40	0.89	17	1
Std. Dev.		Of	
0.26	0.46	62	62
Min		%	
0.06	0.12	27	2
Max			
1.19	2.14		

Table 4 – Summary statistics for all transverse profiles.

Cycloid Dist.	Sinusoid Dist.	Cycloid Subjective	Sinusoid Subjective
Averages Back Lower		Total	
0.53	0.78	2	0
Std. Dev.		Of	
0.29	0.40	12	12
Min		%	
0.14		17	0
Max			
1.19			
Averages Top Lower		Total	
0.42	0.77	0	0
Std. Dev.		Of	
0.14	0.29	10	10
Min		%	
0.20		0	0
Max			
0.70			
Averages Back Upper		Total	
0.24	0.50	6	1
Std. Dev.		Of	
0.13	0.22	12	12
Min		%	
0.11		50	8
Max			
0.62			
Averages Top Upper		Total	
0.26	0.59	4	0
Std. Dev.		Of	
0.15	0.20	8	8
Min		%	
0.06		50	0
Max			
0.50			
Averages Back C		Total	
0.52	1.25	4	0
Std. Dev.		Of	
0.31	0.42	12	12
Min		%	
0.11		33	0
Max			
1.11			
Averages Top C		Total	
0.41	1.41	1	0
Std. Dev.		Of	
0.26	0.28	8	8
Min		%	
0.17		13	0
Max			
0.98			

Table 5 - Summary statistics for transverse profiles, by profile location.

One advantage of a study like this is that comparing profiles to standard curves is a way of deriving some quantitative measure of the variability apparent in the sampled profiles. Looking at the summary data in **table 4** this variability is indicated by the standard deviation values for all the profiles which, compared to the means, are quite high. Note that standard deviation values are high even when the same profile on the same plate are considered (**table 5**). This may be the best evidence that the variability of the data is too great to come to any reliable conclusions concerning the viability of any of the generated curves accurately modeling the profiles.

Another view of the data can be had by considering the range of average distortion values compared to the subjective acceptance scores. Doing so can help qualify the distortion values, which otherwise provide only relative measures of distortion. From **table 3**, the highest average distortion value for either generated curve that was rated as an acceptable approximation to the profile in the subjective scoring is 0.20, and the lowest distortion value that was rated as an unacceptable approximation is also 0.20. Distortion values above 0.20 can be considered to be reliably subjectively scored as *not* indicating a successful modeling, and values below 0.20 can be considered to be reliably subjectively scored as indicating a successful modeling. Given this range, only 17 out of 62 comparisons were considered to show an acceptable modeling of a profile by the curtate cycloid curve.

Tables 5 – 8 represent a first cut attempt at quantifying the high degree of variability shown in the data. The number of samples is too small to draw meaningful conclusions in most cases, but the data are intriguing and may guide future study efforts. The following discussion considers just the subjective evaluation of the curtate cycloid curve. **Table 5** indicates that upper bout profiles for both top and back are well modeled by the curtate cycloid curve for 50% of the profiles, compared to 27% for profiles from all body locations. One possible explanation for this is that this is an area of low age related distortion. Another is that, with the lowest typical length:height ratio of the transverse profiles, profiles at this body location are better modeled by the curtate cycloid curve by simple coincidence. **Table 6** indicates that bassbar side profiles for both top and back show better modeling by the curtate cycloid curve (39% of the profiles). This could also possibly be related to less age and stress related distortion on this side of the instrument. **Table 7** indicates that profiles on the back plate are almost twice as likely to be well modeled by the curtate cycloid (33%) than are the profiles on the top plate (18%). **Table 8** breaks down fit of curtate cycloid model to arching profile by instrument. Here the data do not vary much, from a high of 33% to a low of 17%.

Cycloid Dist.	Cycloid Subjective	Sinusoid Subjective
Averages soundpost side	Total	
0.48	5	0
Std. Dev.	Of	
0.27	31	31
Min	%	
0.11	16	0
Max		
1.19		
Averages bassbar side	Total	
0.33	12	1
Std. Dev.	Of	
0.22	31	31
Min	%	
0.06	39	3
Max		
1.11		

Table 6 - Summary statistics for transverse profiles, by side.

IV. Conclusion

The objective part of the study used a simple measure of average distortion per unit length as a measure of fit between golden age Cremonese instrument arching profiles and three different mathematically generated curves. Two of these curve types (curtate cycloid, sinusoid) were compared to the transverse arching profiles, and all three (curtate cycloid, sinusoid, circular arc) were compared to the limited number of longitudinal profiles available. Each generated curve was sized to the same length and height as the instrument profile it was compared to. The circular arc showed fair fit to the limited number of longitudinal profiles studied. The sinusoidal curve showed a poor fit (0.89 mm^2/mm mean, 0.46 std. dev.) to the transverse profiles. The curtate cycloid showed a better fit to the transverse profiles (0.40 mm^2/mm mean, 0.26 std. dev.), but the high degree of variability shown even for this curve type indicates that it cannot be considered to be a reasonable general model for the actual arching profiles of the instruments considered, given the methods of comparison used in this study. Subjective analysis of the profile comparison diagrams also indicated too much variability to consider this curve type to be a reasonable general model for the arching profiles of the instruments in the study. Readers are encouraged to make use of the comparison figures presented to perform their own subjective analysis. On visual analysis the curtate cycloid curve tended to run mostly above the C bout arching profiles in some cases and mostly below it in others. This may indicate that this curve type represents a good model of average profiles at this position despite high variability of curve shape. This trend was not seen for the upper and lower bout arching profiles. At these locations the curtate cycloid curve tended to run mostly below the arching profiles in all cases. This may indicate that this curve type represents a suboptimal model of average profiles at this position, and suggests that an as yet unidentified curve type may represent a better model.

Cycloid Dist.	Cycloid Subjective	Sinusoid Subjective
Averages back	Total	
0.43	12	1
Std. Dev.	Of	
0.29	36	36
Min	%	
0.11	33	3
Max		
1.19		
Averages top	Total	
0.37	5	0
Std. Dev.	Of	
0.21	28	28
Min	%	
0.06	18	0
Max		
0.98		

Table 7 - Summary statistics for transverse profiles, by plate.

Assuming the desirability of a general mathematical model for the transverse arching profiles of old violins, the percentage of close fits in the subjective analysis provides evidence that the curtate cycloid curve or some quality of it may be a good starting point for development of such a model. Future studies may also find that increasing the size of the population of instruments studied could make evaluations based on plate type, side of instrument and profile location more fruitful.

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Cycloid Dist.	Cycloid Subjective	Sinusoid Subjective
Averages #1	Total	
0.50	1	0
Std. Dev.	Of	
0.24	6	6
Min	%	
0.20	17	0
Max		
0.75		
Averages #2	Total	
0.36	4	0
Std. Dev.	Of	
0.20	12	12
Min	%	
0.11	33	0
Max		
0.70		
Averages #3	Total	
0.49	3	0
Std. Dev.	Of	
0.34	12	12
Min	%	
0.18	25	0
Max		
1.19		
Averages #4	Total	
0.38	3	0
Std. Dev.	Of	
0.19	12	12
Min	%	
0.12	25	0
Max		
0.64		
Averages #5	Total	
0.38	2	0
Std. Dev.	Of	
0.26	10	10
Min	%	
0.11	20	0
Max		
0.86		
Averages #6	Total	
0.34	4	1
Std. Dev.	Of	
0.26	12	12
Min	%	
0.06	33	8
Max		
0.98		

Table 8 - Summary statistics for transverse profiles, by instrument.