Visual Comparison of Ensemble Averaged Transverse Arching Profiles of Golden Age Cremonese Violins and Curtate Cycloid Curves

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Abstract— Previous research indicated that arching profiles of golden age Cremonese violins are not accurately represented by curtate cycloid curves and that a high degree of variability exists among the arching profiles of those instruments. This follow up study generated ensemble averaged arching profiles from size-normalized arching profiles for three profile locations (upper bout, C bout and lower bout) of six violins and visually compared them to mathematically generated curtate cycloid curves. Curtate cycloid curves appear to closely match ensemble averaged profiles at the C bout location only. An attempt was also made to identify other mathematically generated curve types that may offer close approximations to ensemble averaged profiles, using automated curve fitting software. Only polynomials of at least 5th order were identified as candidates offering close approximations. A single Cartesian function approximating the ensemble averaged profiles was identified which may be of use to modern violin makers desiring to make use of these profiles in modern instruments.

I. INTRODUCTION

A previous study [1] comparing plate arching profiles of golden age Cremonese violins to mathematically generated curves showed little correlation between the arching profiles and those curves. That research also indicated a high degree of variability in the shape of profiles from the same plate location but from different instruments, as well as a high degree of variability in how well the mathematically generated curves modeled the actual curves of any of the instruments. A likely source of this observed variability is that the instruments analyzed were simply designed and constructed in an ad hoc fashion. But the possibility exists that the instruments could have been designed with less variable arching profiles and that some of the observed variability is the result of build variation [2], deformation due to the cumulative effects of string tension, deformation due to wear, and deformation due to repair work.

This paper outlines a preliminary investigation of the possibility that plate arching profiles of golden age Cremonese violins may have been modeled by curtate cycloid curves [3,4], as hypothesized by Playfair [5,6]. Although the great variability seen among the arching profiles in [1] seems to preclude this possibility, it is possible that, if that variability is random, any underlying similarity to a cycloid curve could be revealed by removal of the random noise. Ensemble averaging [7] is a technique for improving signal to noise ratio, usually involving the point by point averaging of repeated signal samples. The noise, by definition random, cancels out in the summing revealing the underlying signal.

Ref#	Description
1	Nicola Amati 'Alard'1649
2	Andrea Guarneri 'Conte Vitale' 1676 (viola)
3	Joseph Guarneri Filius Andrea c. 1705
4	Antonio Stradivari 'Viotti' 1709
5	Antonio Stradivari 'Kruse' 1721
6	Guarneri Del Gesu 'Kreisler' 1733

Table 1 – The arching profiles of these golden age Cremonese violins were examined in this study. All profiles were taken from drawings from The Strad posters available from The Strad Library, and are used with permission.

This study makes use of normalized transverse arching profiles at the upper bout, C bout and lower bout positions from six instruments. Graphs of ensemble averaged curves are presented superimposed over their component curves for visual inspection. Curtate cycloid curves were generated based on length and height of the normalized arching profiles and these are also presented with the ensemble averaged curves for visual inspection.

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II. EXPERIMENT

A. Sample Population

Profiles of the five violins and one viola shown in **table 1** were studied. These are the same instruments used in [1]. Selection of instruments and the transverse arching profiles to be analyzed is described in detail in that article and is not repeated here. This study did not include longitudinal arching profiles as these were shown to be poorly approximated by cycloid curves in [1].

B. Data Input, Processing and Display

All instrument arching profiles to be included in the study were scanned into bitmap files at 600 dpi using a calibrated flatbed scanner. The bitmap images were digitized into a series of (x,y) coordinates using **WinDig** digitizing software written by Mr. Dominique Lovy of the Dept. of Physical Chemistry at the University of Geneva and available for free download at http://www.unige.ch/sciences/chifi/cpb/windig.html. The software is designed to digitize printed graphs, and generates a (y) pixel coordinate value for each (x) pixel coordinate value in a graph line. This means the output array of coordinates has a fixed (x) coordinate interval of 1 pixel. At 600 dpi, this translates to an (x) interval of ~0.042mm. It should be noted that the thickness of the lines representing the profiles of the drawings varied and were as much as ~0.42mm thick. The scanning software is designed to hew to the center of a drawn line, but with no indication of the methods and resolutions used to transcribe and print these lines on the posters this value should be taken as an error factor when considering subsequent data comparisons.

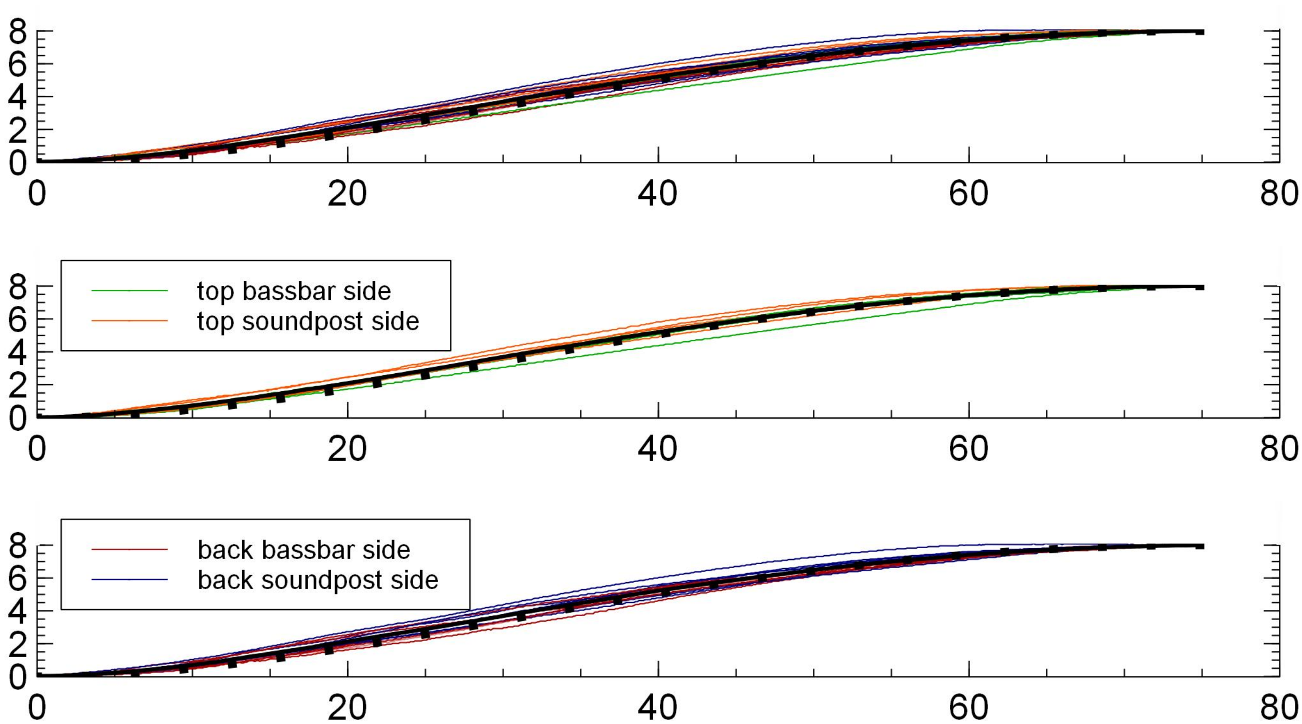

Figure 1 – Transverse arching profiles at the upper bout position for six instruments. The top graph shows profiles for both plates and both sides. The middle graph shows profiles for the belly plate only, while the bottom graph shows profiles for the back plate only. The solid black lines are the ensemble averages, and the dotted black lines are curate cycloid curves. The curtate cycloid curves uniformly under-represent the profile averages at this position.

The coordinate array representation of each arching profile was then input into an Excel spreadsheet for further processing. Array values were offset to origin coordinates (0,0) and then rotated as necessary to compensate for any misalignment between the baseline of the profile and the scanner bed. Coordinate values were then converted from pixels to millimeters. Each full profile was divided in half so each half profile could be processed separately. The right half of the profile was mirrored so that both halves could be viewed in the same orientation. The nadir of the recurve was located for each half profile as the lowest point in the recurve area. The length and height of each half profile was calculated.

The half profiles were grouped by location on the instrument. Three groups were formed for profiles at the upper bout, C bout and lower bout respectively. Average length and height of the violin profiles were obtained for each group and all profiles in a group were scaled to the average length and height values by linear interpolation. Fixed (x) interval was retained, so the horizontal scaling resulted in each curve being represented by the same number of points. This simplifies ensemble averaging math. Each profile was also offset to set the nadir of recurve point at (0,0) and the terminal point at (l,h).

Ensemble average and curtate cycloid curves were calculated for each group. Ensemble averages were calculated using facilities of the software used to plot the graphs (**Veusz 1.13**, a free scientific plotting software package written by Jeremy Sanders and available for free download at http://home.gna.org/veusz/) but could have been performed using spreadsheet software. Curtate cycloid curves were generated as series of points with fixed (x) interval using a JavaScript calculator found at http://www.liutaiomottola.com/formulae/curtate.htm. Each group was further divided by plate (belly, back) and ensemble average and curtate cycloid curves were calculated for each of these subgroups as well.

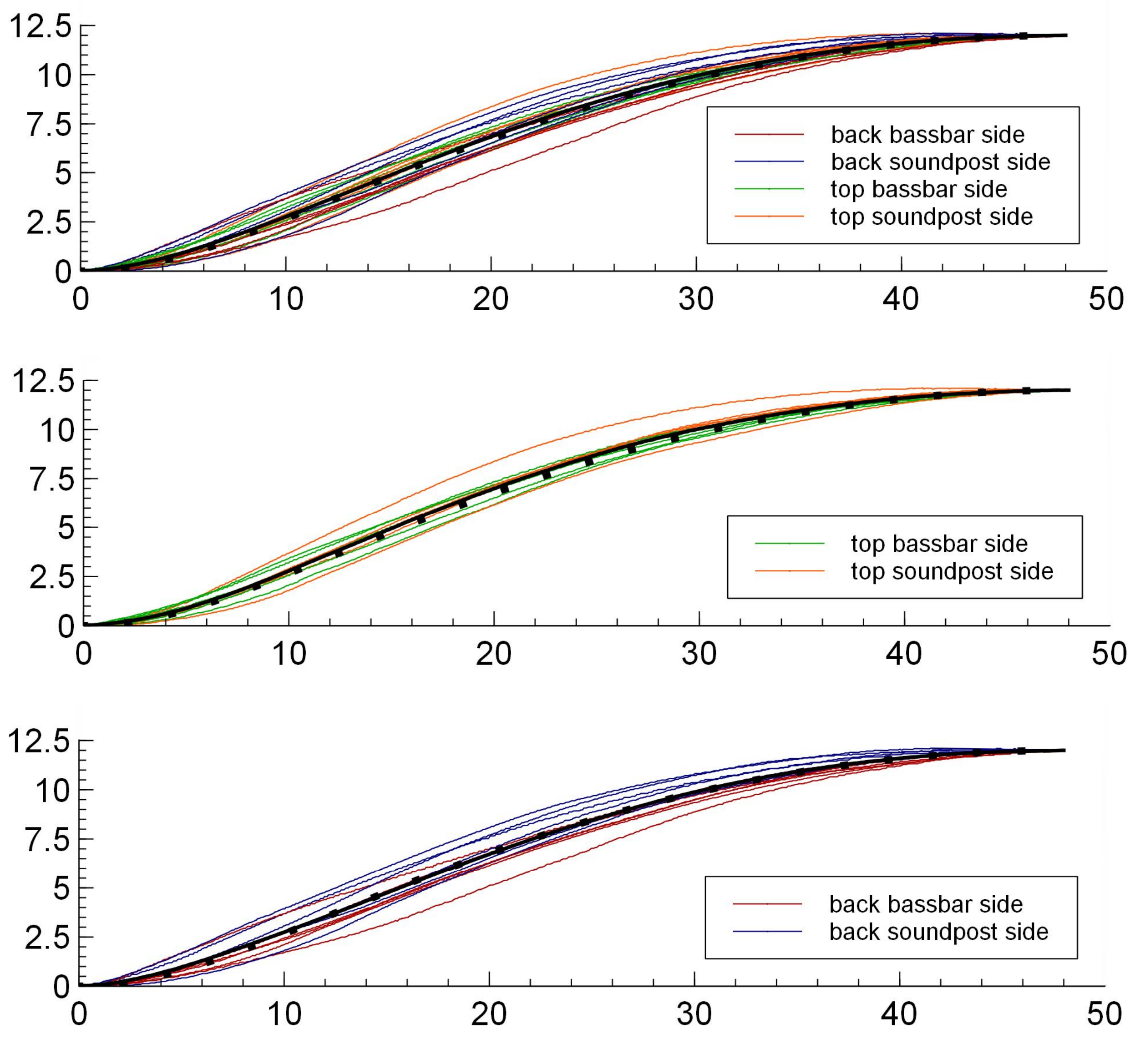

Figure 2 – Transverse arching profiles at the C bout position for six instruments. The top graph shows profiles for both plates and both sides. The middle graph shows profiles for the belly plate only, while the bottom graph shows profiles for the back plate only. The solid black lines are the ensemble averages, and the dotted black lines are curate cycloid curves. The curtate cycloid curves represent the profile averages fairly well at this position, except for the belly plate only profiles.

III. Discussion

A. Visual Comparison

The ability of ensemble averaging to extract a signal from noise depends on the randomness of the noise. Here we consider the likelihood that each of the possible sources of profile shape variation may result in random variation. As mentioned, these sources are build variation, deformation due to the cumulative effects of string tension, deformation due to wear, and deformation due to repair work. It is unlikely that any of these sources would result in truly random variation. Build variation may result in random variability if builders are simply “eyeballing” their work, but if arching templates are used it is unlikely that large random variations would occur. Modern builders using arching templates to duplicate profiles of classic instruments implicitly understand that, because wood carving is a subtractive process, variations between finished work and a template usually mean portions of the finished surface will extend below the template. That is, any variation from the template will be located below the template line. The same is true for deformations due to wear, which can only decrease the depth of surfaces at points of wear, never increase it. It is unlikely that deformations due to the effect of string tension over time would be random, although it is possible that averaging transverse profiles from both sides of the instrument and from both plates would result in these deformations canceling each other out. Deformation due to accumulated repair work may be truly random but evidence to support this is not available.

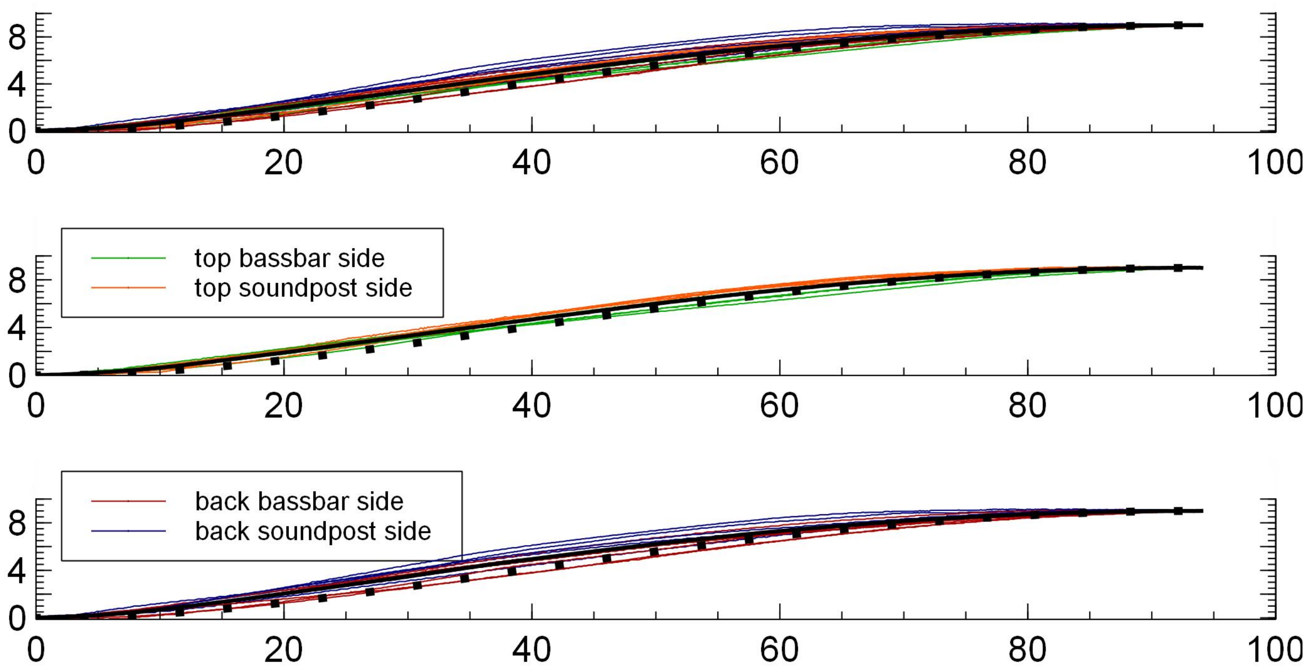

Figure 3 – Transverse arching profiles at the lower bout position for six instruments. The top graph shows profiles for both plates and both sides. The middle graph shows profiles for the belly plate only, while the bottom graph shows profiles for the back plate only. The solid black lines are the ensemble averages, and the dotted black lines are curate cycloid curves. The curtate cycloid curves uniformly under-represent the profile averages at this position.

In summary, that ensemble averaging in this case can reveal any underlying pattern is highly speculative and the figures presented here should be considered accordingly. **Figures 1-3** include comparisons of arching profiles, ensemble averages and curtate cycloid curves at upper bout, C bout and lower bout positions. Comparisons are presented for all profiles at each position and also for belly only profiles and back only profiles at each position. The plate specific graphs are interesting in that profiles tend to group by instrument side for both the upper and lower bout positions. In these positions soundpost side profiles tend to be above the average while bassbar side profiles tend to be below the average, for both belly and back plate. A possible explanation for this in the lower bout position is that installation of successively longer soundposts has over time pushed both plates out on this side. But in that case it might be expected to see this same grouping for the C bout profiles, but that is not the case.

Also interesting is the relationship between the ensemble average curve and the curtate cycloid curve. The cycloid approximates the ensemble average well at the C bout position in the back-only and both plates graphs, but it uniformly under-represents the ensemble average curve at both upper and lower bout positions and in the top-only graph at the C bout position.

B. Additional Analysis

The ensemble averaged curves for this population of instruments were also run through an automated curve fitting program (**CurveExpert Professional 1.5**, http://www.curveexpert.net/) in an attempt to identify simple mathematically generated curves which may closely approximate them. With the exception of polynomials of at least 5th order, no closely matching curves were identified. Each of the ensemble averaged profiles were well defined by 5th order polynomials (worst case correlation coefficient = 0.9999983).

The ensemble averaged curves may be of value as models for modern violin makers. To this end a single function describing all these profiles is highly desirable. The mathematical function search tool **Eureqa Formulize** (available for free download from Nutonian Inc. at http://www.nutonian.com/) was used to identify such a function, using the three transverse ensemble averaged curves as training models. Worst case correlation coefficient for the function is 0.9999128. The Cartesian function is based on curves normalized to range from (0,0 to 1,1). Usage for target curves with origin (0,0) and length ***l*** and height ***h*** is as follows.

$$r = \frac{h}{l} \tag{1}$$

$$x_{norm} = \frac{x}{l} \tag{2}$$

$$y_{norm} = 0.0329 + \tanh\left(2.49x_{norm} + 1.42{x_{norm}}^2 + 75.5x_{norm}r^2 - 0.0329 - 25.2x_{norm}r - 0.424x_{norm}\exp\left(-6.25{x_{norm}}^2\right)\right) \tag{3}$$

$$y = y_{norm}h \tag{4}$$

IV. Conclusion

Visual comparison of ensemble averaged transverse arching profiles of five golden age Cremonese violins and one viola to curtate cycloid curves with the same aspect ratios indicated that curtate cycloid curves did not substantially approximate the averaged profiles. A Cartesian function was identified which describes these ensemble averaged transverse arching profiles and may be of use to modern violin makers wishing to use these as model curves.

V. Acknowledgments

The publishers of *The Strad* were kind enough to permit use of the arching profiles from instruments in *The Strad* poster series. Their contribution is gratefully acknowledged.

VI. Bibliography

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