Modeling and Optimizing of the First Guitar Mode

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Abstract— The first peak in the frequency response function (a ratio between the sound pressure at 1 m from a guitar and mechanical impulse at the bridge) is assumed to be a Fast Fourier Transformation (denoted as p') of sound pressure changes due to damped oscillation of a virtual and hybrid mechanical-acoustic system (m-b-k-A). This consists of a mass (discrete mass m), damper (coefficient of viscous damping b) and spring (stiffness k) which are variables, and a massless membrane with area A (surface A) which is a constant. Depending on its position, a 20 gram weight placed on the top board variously affected amplitude, frequency, and damping of p'. Thus, the position of the weight influences the system dynamics, which is defined through the mechanical quantities m, b and k. A high degree of inverse proportionality between the first guitar mode intensity (or amplitude of p') on the one hand and coefficient of viscous damping b on the other hand was measured. A consequence of this feature works well in modeling and optimizing of the first guitar mode [1, 2]: At coordinates on a guitar top for which the weight results in relatively high b the brace results in relatively low b and consequently in relatively high intensity and low damping of the analyzed mode.

I. INTRODUCTION

In general, each wooden part of a musical instrument, which contributes to sound radiation, should be considered a unique element [3]. The reason lies in the non-homogeneity of wood, which means that different guitar soundboards with the same shape require a different arrangement of braces in order to achieve optimal results. Therefore, the focus of this paper is on the procedure for brace optimizing [1] that should aid the luthier to place the braces according to the actual acoustic properties of the instrument during its construction [4].

An experiment where the excitation was performed by a mechanical impulse at the bridge, and the response was a sound pressure 1 m from the guitar [1, 5] showed that the first resonant peak corresponds to a normal (independent) radiating mode. This is an interaction between the top and back plate and the air inside the resonance box [6, 7]. A comparison between good and bad classical guitars showed that higher amplitude, lower or equal damping, and lower frequency of the first resonant peak in the FRF are significant for the good guitars in comparison to the bad ones [5, 8, 9]. Therefore it is assumed that the amplitude of this peak is strongly correlated with the acoustic response of the top and back plate. Low amplitude of the peak indicates low initial loudness of a guitar tone. This was confirmed by measurements which showed that the amplitude of the first resonant peak in the FRF could be approximately proportional to the loudness of guitar tones (at least for those on open strings E, A and probably also D) with duration of 0.256 second and recorded 0.5 second or sooner after the string excitation [1, 5]. More precisely, the amplitude of the first resonant peak correlated with the initial loudness for all frequencies which lie in the vicinity of this peak [7]. Finally, an increase in amplitude and a decrease in damping factor and frequency of the first resonant peak in the FRF of a guitar can be seen as the aim of a procedure for guitar quality optimization.

This paper presents modeling of the first resonant peak in the FRF by an oscillating system consisting of a mass, viscous damper, spring, and sound-radiating surface. Such a virtual, hybrid mechanical-acoustic system is based on the idea about guitar modes modeling with several systems consisting of a piston, spring and damper, introduced by Richardson [10]. The second part of the paper describes the experiment where a 20 gram weight was placed on the top board and its effects on the mass, coefficient of viscous damping, and stiffness of the oscillating system were observed. It seems that based on the establishment of these effects it is, with high certainty, possible to predict in terms of tone loudness an optimal position of the brace on the top of the guitar [1, 2]. In contrast to methods which are based on optimization of free-supported soundboards [11, 12] the presented model serves for an optimization of brace position for the assembled guitar with or without some of its braces. In this approach the changes in board fixation after its optimization and the corresponding unpredictable effects are avoided. However, the physical explanation of the correlation between the model and guitar design features is not so clear.

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II. Methods and Results

An arrangement for measuring the FRF of a guitar and a place of guitar excitation are defined in our recent paper [1]. Briefly, a mechanical impulse at the bridge was delivered by an accelerometer, which also enabled a quantification of the impulse. A response signal was measured by a microphone at one meter from a guitar in the sound isolated chamber laden with foam rubber in the inner side. Figure 1, which represents a starting point for this paper, shows two FRFs for the same guitar: (i) with uncovered soundhole and (ii) with soundhole covered with foam rubber. One can see that because of the covered hole the first resonant peak practically disappeared from the FRF.

The FRF of a guitar is defined as a ratio between the sound pressure and mechanical impulse in the frequency domain. Let the first resonant peak in this FRF represent Fast Fourier Transformation (*i.e.*, FFT) of sound pressure *p*'(*t*) which is a result of the damped oscillation of the virtual system shown in Figure 2(a), and let the FFT of *p*'(*t*) be denoted as *p*'. This system, denoted as system (*m*-*b*-*k*-*A*), is a hybrid mechanical-acoustic system consisting of a mass (discrete mass *m*), damper (coefficient of viscous damping *b*), spring (stiffness *k*) and massless membrane with area *A* (surface *A*). To remove the probable effects of the neighboring modes, first an average impulse response function of the measured FRF was calculated, then filtered with a narrow-band inverse Chebyshev filter, and finally transformed into a frequency domain. The filter bandwidth was 1 Hz, which means its absolute frequency ranged between the frequency of *p*' +/- 0.5 Hz. For the frequencies between 70 and 120 Hz the filter's gain factor was approximately 0.5 which means a nearly constant influence of the filter on *p*'. A result of an inverse FFT of the FRF was a one-sided impulse response [13, 14] which means two times larger amplitudes in comparison to a two-sided impulse response. The measured FRF was always two-sided, thus the gain factor of the filter (approximately 0.5) did not significantly influence the amplitude of the filtered *p*'. The properties of the filter were: Sampling frequency - 16 kHz, coefficient of attenuation - 1 dB, coefficient of order - 3. A typical result of the usage of this filter after the measurement of FRF of a guitar and its inverse Fourier Transformation is shown in Figure 2(b).

To define the system (*m*-*b*-*k*-*A*) the following equation which expresses the sound pressure in a spherical sound wave generated by a small vibrating body is introduced [15]:

$$p(t) = \frac{S \cdot r \cdot c \cdot z}{4p \cdot r} \sin\ z(c \cdot t - r)\,, \tag{1}$$

where *S* is maximum rate of air emission of the small source, *r* density of the air, *c* velocity of sound, $z = 2p/l$ (l is a wavelength), *r* distance from the sound source, and *t* time.

As already defined, the distance between the guitar and the microphone was 1 m. The diameter of the soundhole with area *A* (surface *A*) which can be seen as a source of sound pressure is relatively small in comparison to this distance. Therefore, equation (1) is a good starting point for the description of the damped sound pressure *p*'(*t*). Surface *A* vibrates according to the impulsively excited system consisting of a mass, damper, and spring. Figure 2(a) shows that a massless membrane with

List of Symbols

A	area of sound radiating surface
b	coefficient of viscous damping
b'	coefficient of viscous damping of system (*m-b-k-A*) (without weight)
b(*x*,*y*)	coefficient of viscous damping of system (*m-b-k-A*) [weight on (*x*,*y*)]
b(*r*)	average coefficient of viscous damping of system (*m-b-k-A*)
c	velocity of sound
f	frequency
f_{0d}	frequency of natural damped oscillation of system (*m-b-k-A*)
f'_{0d}	frequency of natural damped oscillation of system (*m-b-k-A*) (without weight)
$f_{0d}(x,y)$	frequency of natural damped oscillation of system (*m-b-k-A*) [weight on (*x*,*y*)]
I	mechanical impulse
k	stiffness
k'	stiffness of system (*m-b-k-A*) (without weight)
k(*x*,*y*)	stiffness of system (*m-b-k-A*) [weight on (*x*,*y*)]
k(*r*)	average stiffness of system (*m-b-k-A*)
m	discrete mass
m'	mass of system (*m-b-k-A*) (without weight)
m(*x*,*y*)	mass of system (*m-b-k-A*) [weight on (*x*,*y*)]
m(*r*)	average mass of system (*m-b-k-A*)
P	peak amplitude of sound pressure *p*'
P'	peak amplitude of sound pressure *p*' (without weight)
P(*x*,*y*)	peak amplitude of sound pressure *p*' [weight on (*x*,*y*)]
p'	sound pressure due to oscillation of system (*m-b-k-A*) - FD
p'(*t*)	sound pressure due to oscillation of system (*m-b-k-A*) – TD
P_a	approximation of peak amplitude of *p*'
$p_u(t)$	undamped sound pressure
Q	quality factor of resonance
r	distance from the sound source
r1, r2	brace positions
S	maximum rate of air emission
t	time
(*x*,*y*)	weight's position
X(*t*)	displacement of surface *A*
$\dot{X}$	velocity of surface *A*
$\ddot{X}$	acceleration of surface *A*
X_s	initial amplitude of oscillation
$\dot{X}_s$	initial velocity of oscillation
d	viscous damping factor of system (*m-b-k-A*)
d'	viscous damping factor of system (*m-b-k-A*) (without weight)
d_a	approximation of viscous damping factor of system (*m-b-k-A*)
d (*x*,*y*)	viscous damping factor of system (*m-b-k-A*) [weight on (*x*,*y*)]
l	wavelength
r	density of air
w_0	circular frequency of natural undamped oscillation of system (*m-b-k-A*)
w_{0d}	circular frequency of natural damped oscillation of system (*m-b-k-A*)

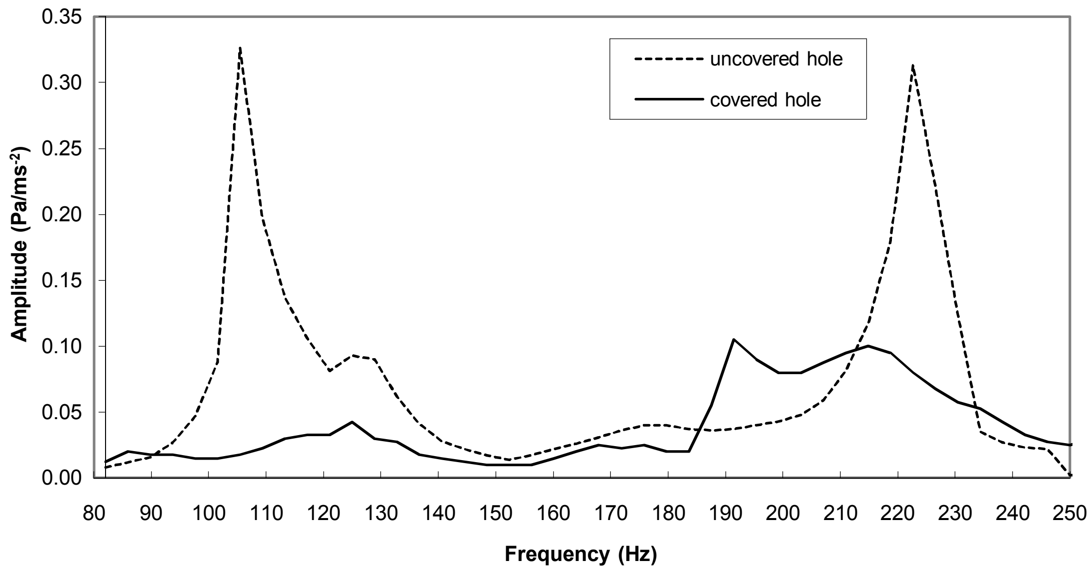


Figure 1 - Amplitude diagram of FRF for the tested guitar.

area *A* is rigidly connected to the mass *m*. Thus, *p*'(*t*) is directly dependent on *m*, *b* and *k* which determine the natural damped oscillation of system (*m*-*b*-*k*-*A*). If such a system is excited by an impulse (impulsive excitation of a guitar at the bridge) then the equation of motion for surface *A* is [16]:

$$m\ddot{X}(t) + b\dot{X}(t) + kX(t) = 0\,, \tag{2}$$

where $\ddot{X}$ and $\dot{X}$ are acceleration and velocity of surface *A*, respectively. Solution of this equation is [16]:

$$X(t) = \mathrm{e}^{-d\, w_0\, t}\left(X_s \cos w_{0d}\, t + \frac{\dot{X}_s + d w_0 X_s}{w_{0d}} \sin w_{0d}\, t\right), \tag{3}$$

where X_s and $\dot{X}_s$ are initial amplitude and initial velocity of oscillation in the moment of cessation of impulse disturbance, w_{0d} is a natural frequency of the damped oscillation $w_{0d} = \sqrt{1-d^2} \cdot w_0$, w_0 is a natural frequency of the undamped oscillation $w_0 = \sqrt{k/m}$, and d is the dimensionless viscous damping factor $d = b/2\sqrt{km}$ [16].

Considering that (i) velocity of surface *A* before the action of the mechanical impulse *I* is 0 and after its acting $\dot{X}_s$, (ii) $\dot{X}_s = I/m$, and (iii) that the analyzed system has a well known behavior, we can write [16]:

$$X(t) = e^{-d\, w_0\, t}\left(\frac{I}{m w_{0d}} \sin w_{0d} t\right). \tag{4}$$

Considering that $z = 2p/l$ and $l = c/f$ (f is frequency of sound), equation (1) gives for a case of the undamped oscillation:

$$p_u(t) = \frac{S \cdot r \cdot \frac{c \cdot 2p \cdot f}{c}}{4p \cdot r} \sin \frac{2p \cdot f}{c}(c \cdot t - r)\,. \tag{5}$$

In this equation $p(t)$ from equation (1) was substituted with $p_u(t)$ which indicates undamped sound pressure. According to Figure 2(a), S in m^3/s depends on surface A and maximal velocity of oscillation ($\dot{x}_{max}$) of this surface [15]:

$$S = A \cdot \dot{x}_{max} \,. \tag{6}$$

Introducing equation (6) into equation (5), we obtain

$$p_u(t) = \frac{A \cdot \dot{x}_{max} \cdot r \cdot f}{2 \cdot r} \sin \frac{2p \cdot f}{c}(c \cdot t - r) \,. \tag{7}$$

From equation 4 it is evident that damping of the oscillation of system (*m*-*b*-*k*-*A*) and thus damping of $p_u(t)$ can be expressed by $e^{-dw_0 t}$. Frequency f is actually a natural frequency of the damped oscillation of system (*m*-*b*-*k*-*A*). After substituting $p'(t)$ for $p_u(t)$ in equation (7) we obtain:

$$p'(t, w_{0d} = const) = e^{-dw_0 t}\left(\frac{A \cdot \dot{x}_{max} \cdot r \cdot w_{0d}}{4p \cdot r} \sin \frac{w_{0d}}{c}(c \cdot t - r)\right) \tag{8}$$

and

$$w_{0d} = 2p \cdot f_{0d} \,, \tag{9}$$

where f_{0d} is frequency of natural damped oscillation of system (*m*-*b*-*k*-*A*) in Hz. From equation (4) we can calculate the first derivative of the amplitude of oscillation which means the velocity of oscillation of system (*m*-*b*-*k*-*A*). Of course maximal velocity of oscillation of surface A occurs at the beginning of oscillatory motion of system (*m*-*b*-*k*-*A*):

$$\dot{x}_{max}(t = 0) = \frac{I}{m} \,. \tag{10}$$

Finally, sound pressure $p'(t)$ can be written as

$$p'(t, w_{0d} = const) = p'(t) = e^{-dw_0 t}\left(\frac{A \cdot I \cdot r \cdot w_{0d}}{4p \cdot r \cdot m} \sin \frac{w_{0d}}{c}(c \cdot t - r)\right). \tag{11}$$

Frequency resolution of the FRF plots was approximately 0.976 Hz. The duration of input and output signal was 1.024 seconds, sampling frequency was 16 kHz and number of acquired discrete points was 16384. The algorithm for determination of m, b and k was relatively simple. First, from p' (measured) three quantities were read: frequency f_{0d} (w_{0d}), peak amplitude P and viscous damping factor d. After this, a modeling of function $p'(t)$ from equation (11) was performed [$p'(t)$ depending on m, b and k, while A is a constant]. Searching for a suitable combination of m, b and k was done on the basis of minimal deviation between the measured P and d on the one hand, and calculated P_a and d_a on the other hand. The latter two are an approximation of peak amplitude and viscous damping factor of p'. In the last step of modeling the difference between P and P_a was less than 1%. In addition, the difference between d and d_a was less than 3% which indicates a relatively accurate modeling of p' by a system (*m*-*b*-*k*-*A*).

Experiment with a covered soundhole showed that the area of surface A can be considered as constant. Because excitation at the bridge was always the same [5] and because p' is derived from a FRF, quantity I can be considered as constant, as well. System (*m*-*b*-*k*-*A*) is a virtual system, thus an accurate definition of A and I is insignificant. Therefore a value of product $A \cdot I$ was arbitrary determined as 0.01 kg·m^3/s, however any other value (positive number) would enable the following calculations equally well. In the following measurements this resulted in a mass m about 1 kg, which is very roughly a mass of both resonant boards. The second reason for the insignificance of the

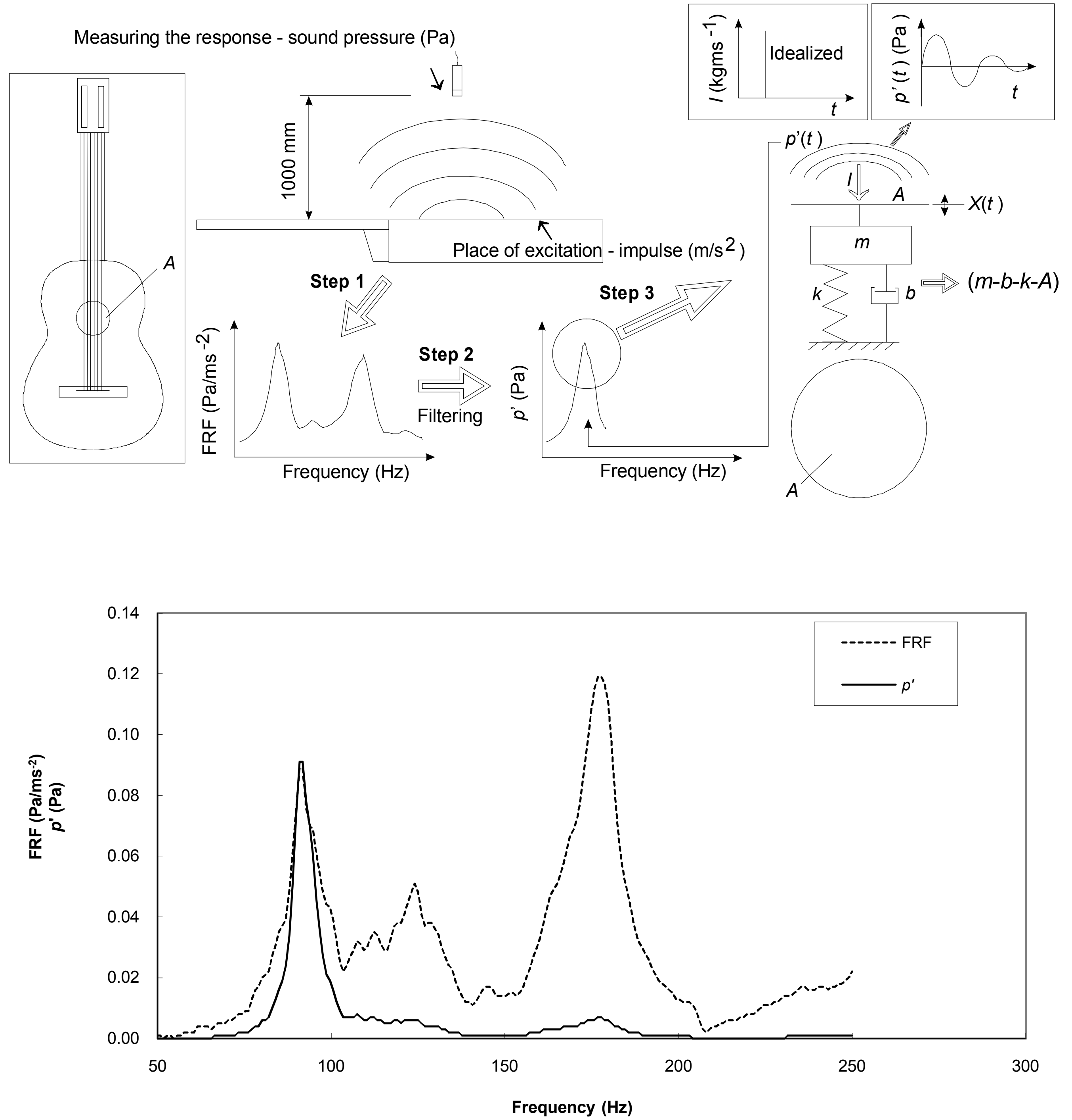
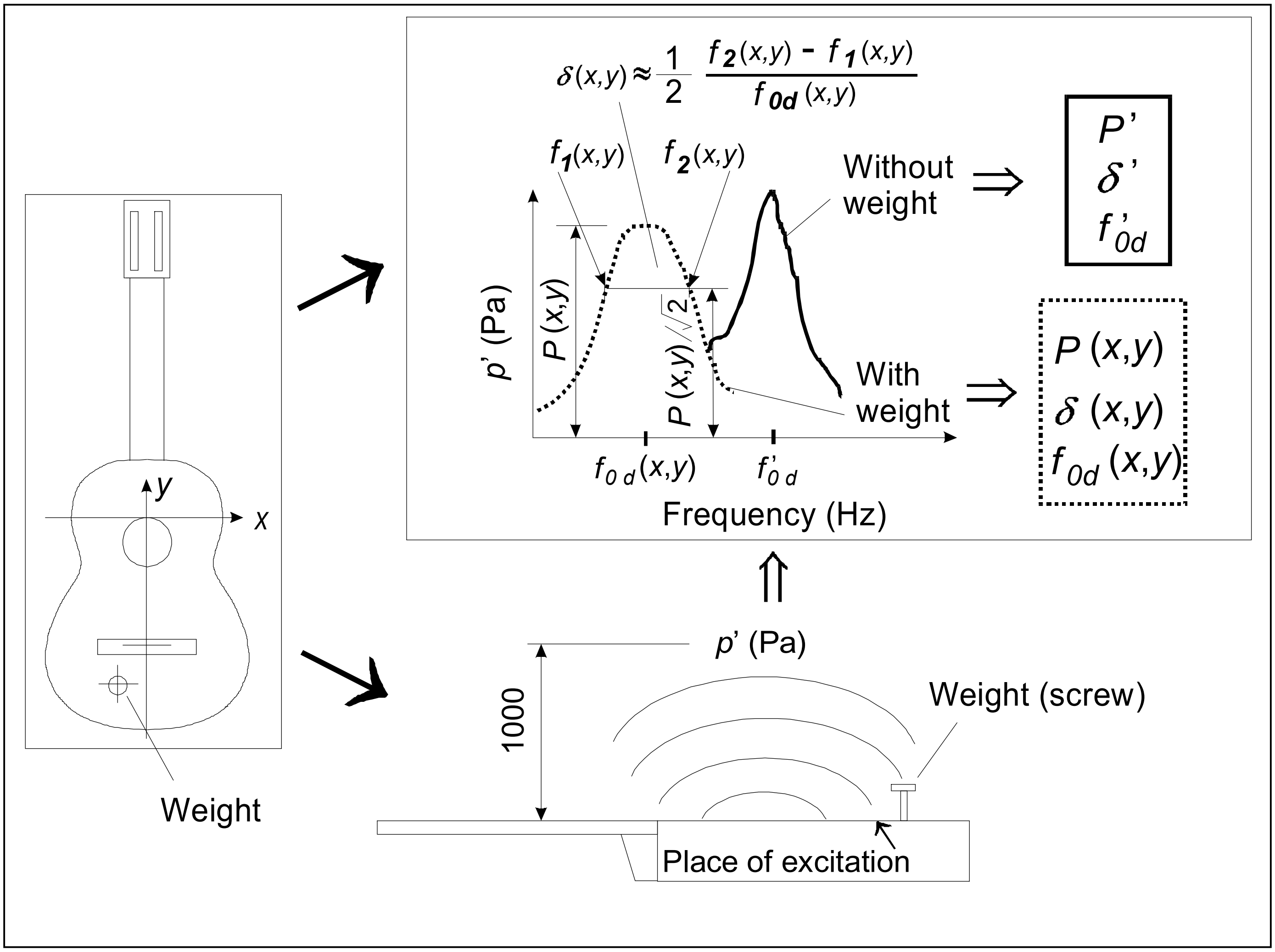


Figure 2 – Definition of *p*': (a, above) Hybrid system (*m-b-k-A*) as a source of *p*'(*t*) and *p*', respectively (*I* is mechanical impulse); (b, below) The resulting *p*'.

absolute value of product $A{\cdot}I$ is that the following analysis depended on the differences between the *m*, *b* and *k* rather than on the *m*, *b* and *k* themselves.

In the analysis that follows, a theoretical basis for a method of optimizing the position of braces on the resonant board of a guitar is given. The method is based on the idea about returnable changes performed on the assembled guitar [17]. These changes were performed by putting the weight onto the top resonant board (*i.e.*, board) as shown in Figure 3. The position of the weight affected *p*', and thus also quantities *m*, *b* and *k*.

Let the places with a weight on the board have co-ordinates (x,y). Putting the weight down on the board is physically not the same as increasing the mass of the board. Thus, the effect of the weight on the board can be denoted as an increase of so called quasi mass of the guitar. The mass of the weight was 20 g which was based on the rule accepted in measurements of dynamic behavior of a light structure. The mass of an accelerometer should be at least 10 times smaller than a mass of the measured structure [13]. In contrast with such measurements, some influence of the weight on the modal behavior of the board (guitar) was desirable. The mass of the board without braces was



approximately 200 g, thus a 20-gram weight fulfills this condition.

The weight in position (x,y) affects P, d and f_{0d} which can therefore be denoted as $P(x,y)$, $d(x,y)$ and $f_{0d}(x,y)$. Quantities P, d and f_{0d} for the board without the weight may be denoted as P', d' and f'_{0d}. An example of determining $P(x,y)$, $d(x,y)$ and $f_{0d}(x,y)$ is shown in Figure 3. By analogy P', d' and f'_{0d} were determined from p' for the guitar board without the weight. Evidently m, b and k were calculated from P, d and f_{0d}. By analogy, from $P(x,y)$, $d(x,y)$ and $f_{0d}(x,y)$ the corresponding $m(x,y)$, $b(x,y)$ and $k(x,y)$, which represent m, b and k for a weight at a position (x,y), were calculated. In the same way from P', d' and f'_{0d} the corresponding m', b' and k', which represent m, b and k for a guitar without the weight, were calculated (see also Figure 3).

Figure 4 shows a test guitar with a top resonant board without two large and two small braces near the hole and the co-ordinates of the weight ($x = x_i, y = y_j$) where i=1, 2 … 13, j =1, 2. These co-ordinates fit an area of a possible position of the large cross brace. An example of an usual shape and position of this brace is also shown in Figure 4. For each position of the weight, p' was measured and after that, as indicated above, $m(x = x_i, y = y_j)$, $b(x = x_i, y = y_j)$ and $k(x = x_i, y = y_j)$ were estimated, see Figure 5 (m'=0.345 kg, b'=9.379 $Nm^{-1}s$, and k'=106711.3 N/m). As indicated in Figure 4, let the 13 positions of the weight (x=x_i,y_1=7 mm) form line r1. By analogy, the positions of the weight (x=x_i,y_2=14 mm) form line r2. Lines r1 and r2 are also possible positions of the large brace. Figure 6(a) shows the

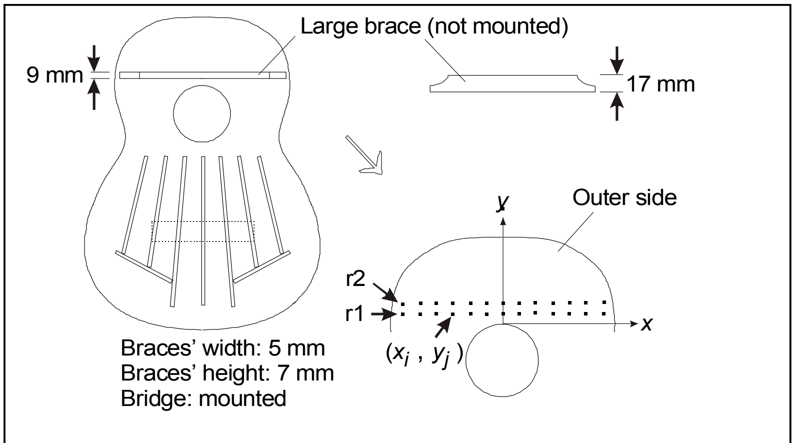


Figure 4 - Tested top board and a possible position of a large brace.

results of experimentation by changing the large brace position. Since the top board was a constant during these experiments both the top and brace were varnished in order to ease the ungluing of the brace and to change its position. It is evident that the brace on r1 resulted in a higher amplitude, smaller damping and nearly the same frequency of *p*' in comparison to the brace on r2. With respect to the correlation between the tone loudness and characteristics of the *p*' (see section 1 and [5]) brace position r1 was denoted as a more favorable brace position in comparison to r2. An additional experimentation showed that for the brace on a position defined by the places (*x*,*y*) with relatively high ratio *b*(*x*,*y*)/*m*(*x*,*y*)·*k*(*x*,*y*), *p*' is more favorable in comparison to the position defined by places where this ratio is relatively low. For both tested brace positions r1 and r2, the ratio *b*(*x*,*y*)/*m*(*x*,*y*)·*k*(*x*,*y*), and *p*' is shown in Figure 6(b). Indeed, a comparison of lines r1 and r2 shows that for the majority of weight positions *b*(*x*,*y*)/*m*(*x*,*y*)·*k*(*x*,*y*) is higher (or not significantly lower) for the places (*x*,*y*) on the guitar top which form line r1.

Several hundreds of measurements with weight and optimization of 22 different braces on the various guitar tops confirmed the correlation between the optimal brace position (in terms of intensity of the first guitar mode) formed by places (*x*,*y*) on the one hand, and relatively high *b*(*x*,*y*)/*m*(*x*,*y*)·*k*(*x*,*y*) measured on these places on the other hand. More precisely, gluing of a brace on positions with relatively high ratio *b*(*x*,*y*)/*m*(*x*,*y*)·*k*(*x*,*y*) always resulted in a relatively low ratio *b*'/*m*'·*k*'. Table 1 shows these correlations for most typical 10 of 22 experiments with different braces on different areas above and below the soundhole of various guitar tops with equal shape. In each experiment only two brace positions indicated as r1 and r2 were analyzed, as in experiment shown in Figure 4. In addition, in each experiment the same brace was subsequently glued on these two positions. Figure 7 (a, b) shows brace positions r1 and r2 on guitar tops as well as shape of the braces. For these brace positions averages of single values *b*(*x*,*y*), *m*(*x*,*y*) and *k*(*x*,*y*) obtained during experimentation with the weight were calculated and indicated as *b*(*r*), *m*(*r*) and *k*(*r*), respectively where *r* is r1 and r2. Ratio *b*(*r*)/*m*(*r*)·*k*(*r*) as well as ratio *b*'/*m*'·*k*', and quantities *P*', *d* ' and f_{0d}' for a situation before and after the brace gluing are also indicated in Table 1. A

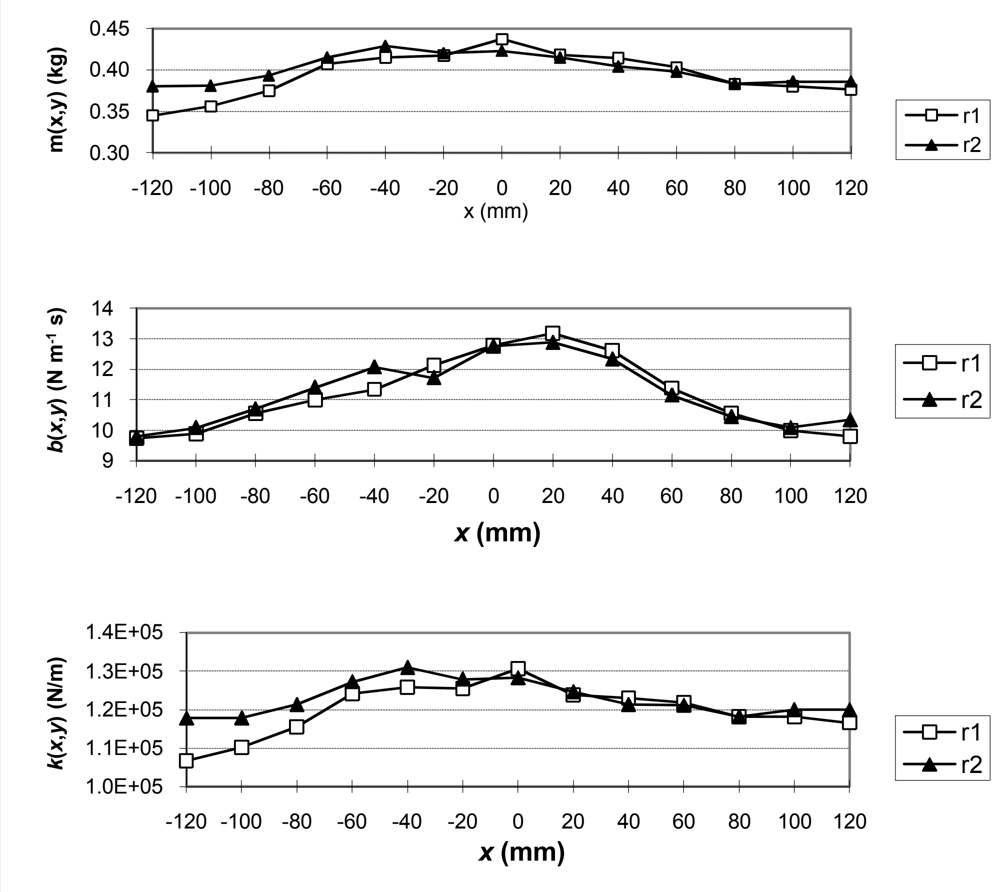


Figure 5 - *m*(*x*,*y*), *b*(*x*,*y*) and *k*(*x*,*y*) as function of the weight's position (*y*∈r1, r2).

physical meaning and explanation of correlations from Table 1 will be discussed in the following section.

III. Discussion

Figure 8 shows more than one hundred randomly selected results from experiments with the weight positioning and brace gluing. It is evident that there is a significant inverse proportionality between the peak amplitude of *p*' on the one hand and coefficients of viscous damping *b*' and *b*(*x*,*y*), respectively on the other hand. This correlation is clearly in force for measurements with and without the weight, as well as for situations before and after the brace gluing. In our case (impulse excitation of a guitar) we can see from equations (6) to (11) that amplitude of *p*' is proportional to the velocity of a massless membrane with surface *A*. This velocity is proportional to w_{0d} and inversely proportional to *m*. In other words, in the presented system (*m*-*b*-*k*-*A*) we do not presume any significant dependence of amplitude of *p*' on coefficient of viscous damping *b*: The effect of damping factor d (which depends also on *b*) on w_{0d} is negligible due to low magnitudes of d (d = 0.015 to 0.040). However, for a one-mass mechanical system of one degree of freedom, excited by a sinusoidal driving force, the following is true [15]: Velocity of mass increases as the magnitude of the dissipative or resistive element decreases for a certain magnitude of a driving force. In our case, both *b*' for situation after brace gluing and *b*(*x*,*y*) for situation with weight positioning can be defined as a resistive element or impedance of the analyzed guitar mode [6, 15]. Based on Figure 8, we can conclude that modeling of the first guitar mode with impulsively excited system (*m*-*b*-*k*-*A*) enables a definition of impedance of this mode. These findings strongly support a physical sense of system (*m*-*b*-*k*-*A*) in spite of the fact that it represents a very rough (indirect) model of modal behavior of a guitar. We can also say that coefficient of viscous damping *b* of system (*m*-*b*-*k*-*A*) is a good measure for mechanical (acoustical) impedance of the analyzed mode regardless of a type of excitation.

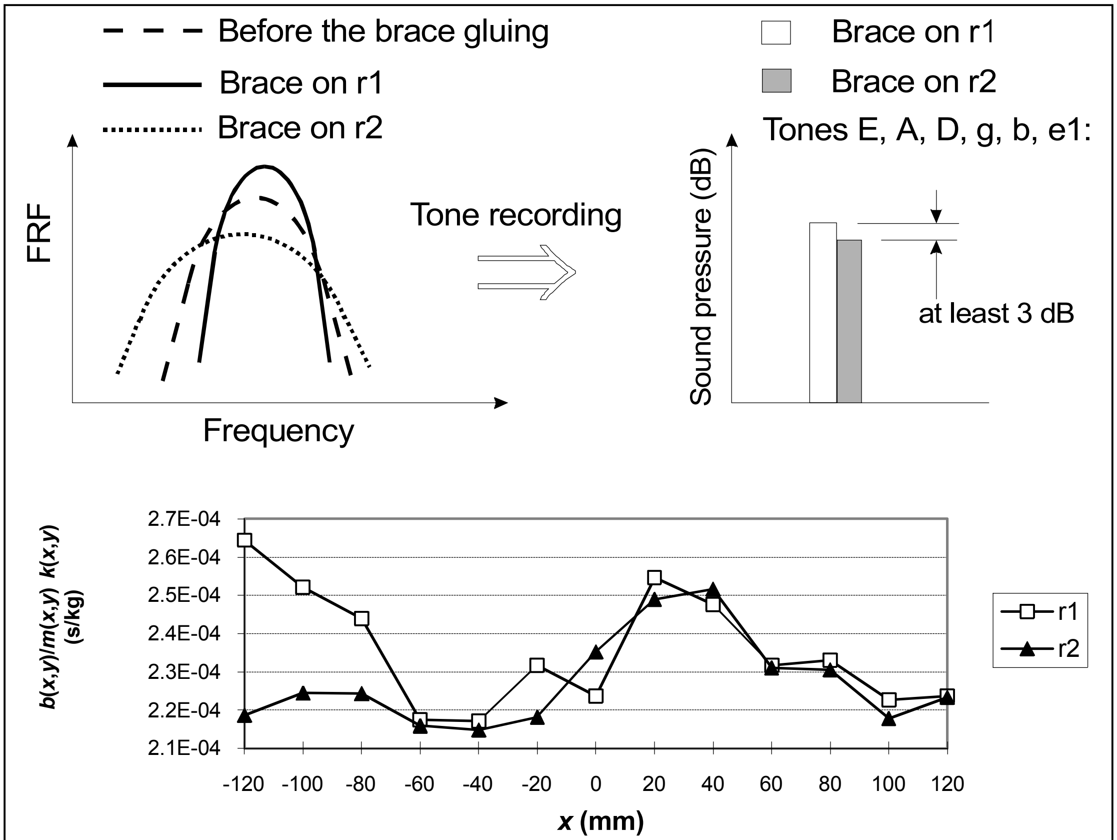


Figure 6 - The effect of the two different brace positions (schematically): (a) The first resonant peak in the FRF of the test guitar and initial intensity of tested tones; (b) Ratio *b*(*x*,*y*)/*m*(*x*,*y*)·*k*(*x*,*y*) where *y*∈r1, r2.

As indicated at the end of section 2, there is a strong correlation between the effect of the weight and the effect of the brace. In all of 22 experiments with brace gluing this was reflected through a certain correlation between ratios *b*(*r*)/*m*(*r*)·*k*(*r*) and *b*'/*m*'·*k*'. As a rule, when the former was relatively high (low), the latter was relatively low (high). From a theory of one-mass mechanical systems a quality factor of a resonance *Q* [13] is defined as a very close approximation to 1/2d. Next, d is proportional to *b*(*r*)/*m*(*r*)·*k*(*r*) for measurements with the weight and to *b*'/*m*'·*k*' for measurements after the brace gluing (*i.e.*, for a situation after the brace gluing $d = b'/2\sqrt{m'\cdot k'}$). Logically, a higher *Q* (which also means a higher amplitude of *p*' [15]) means lower d, thus relatively low *b*' in comparison to *m*'·*k*'. As we can see from Table 1, this explains why a relatively high *P*' (amplitude of *p*') is correlated to a relatively low d of the first guitar mode.

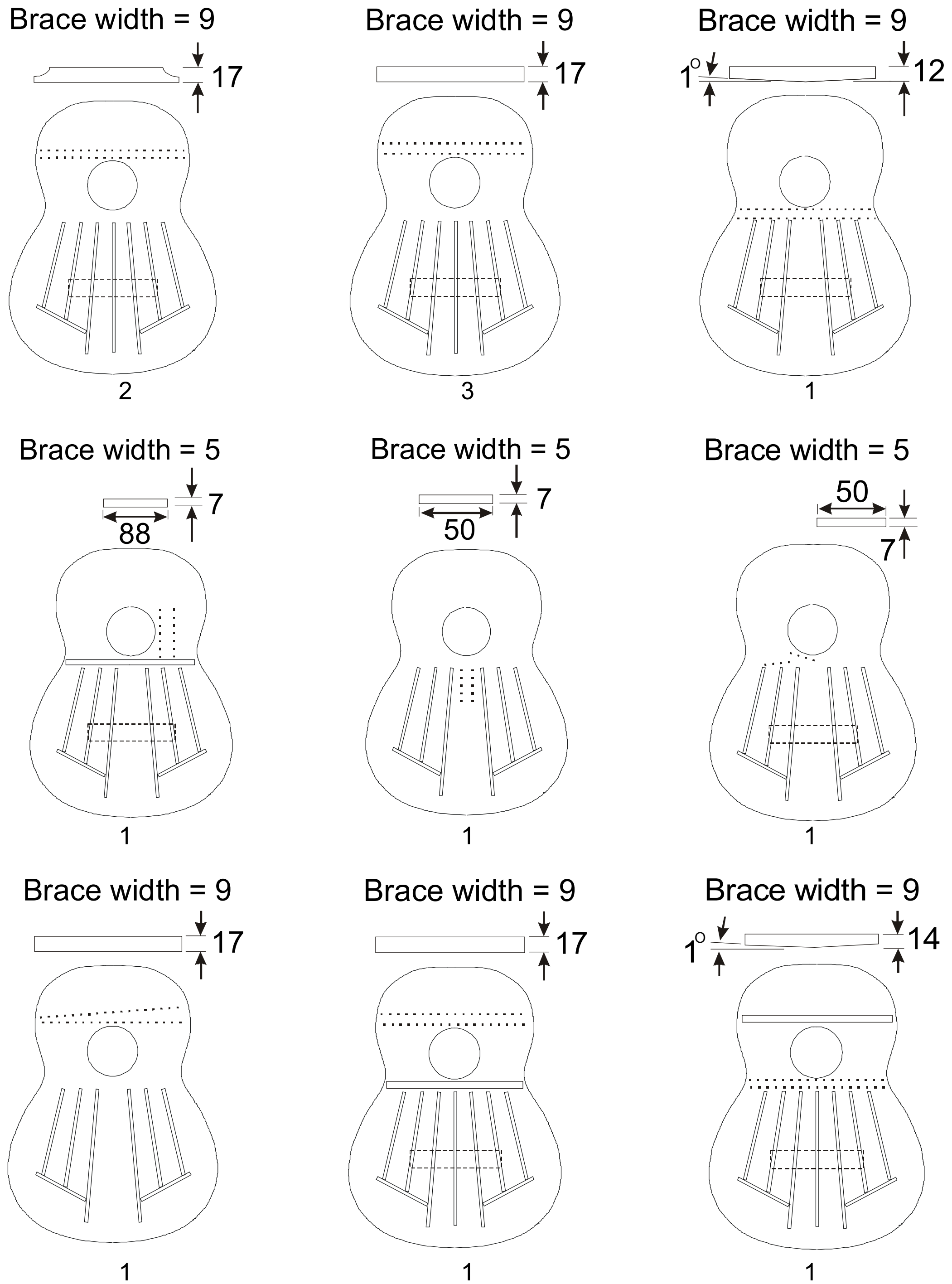


Figure 7a - Tested guitar tops, brace positions (doted lines) and tested braces. Number below each configuration indicates quantity of tested tops and braces. All dimensions are in millimeters. If glued on the top board, the bridge is indicated by a dashed line.

According to Figure 8 the meaning of $b(x,y)$ and b' which mean dissipation (impedance) is well grounded, which is not true for $m(x,y)\cdot k(x,y)$ and $m'\cdot k'$. It is evident that the weight influences the modal behavior of the top and therefore also of the *top board-air-back board* triplet, in a complex way. Thus, the effect of the weight can be seen as both "indirect" and a "direct". The changes in modal shape, damping, frequency and amplitude of the triplet can be considered as the consequences of both effects whose interaction is complex and unknown. The changes of the modal behavior of the triplet can result in any changes of the mass, stiffness, and coefficient of viscous damping of the "pumping mode" which is experimentally proven. More precisely, experiments showed that (i) $m(x,y)$ and $k(x,y)$ can be higher, equal or lower than m' and k' respectively, and (ii) $b(x,y)$ can be higher or equal than b', in general.

The “direct” effect of the weight theoretically results only in an increase of modal mass of the triplet. However, the measurements of $m(x,y)$ (higher, equal or lower than m') shows that this effect is not always evident. But since the frequency of the first peak in the FRF of a guitar always decreased after putting the weight on the top, the “direct” effect of the added mass was actually always prominent: In spite of the complex changes of the modal behavior of the triplet, the added mass of the weight resulted in a larger influence on the modal mass than on the modal stiffness which is physically acceptable.

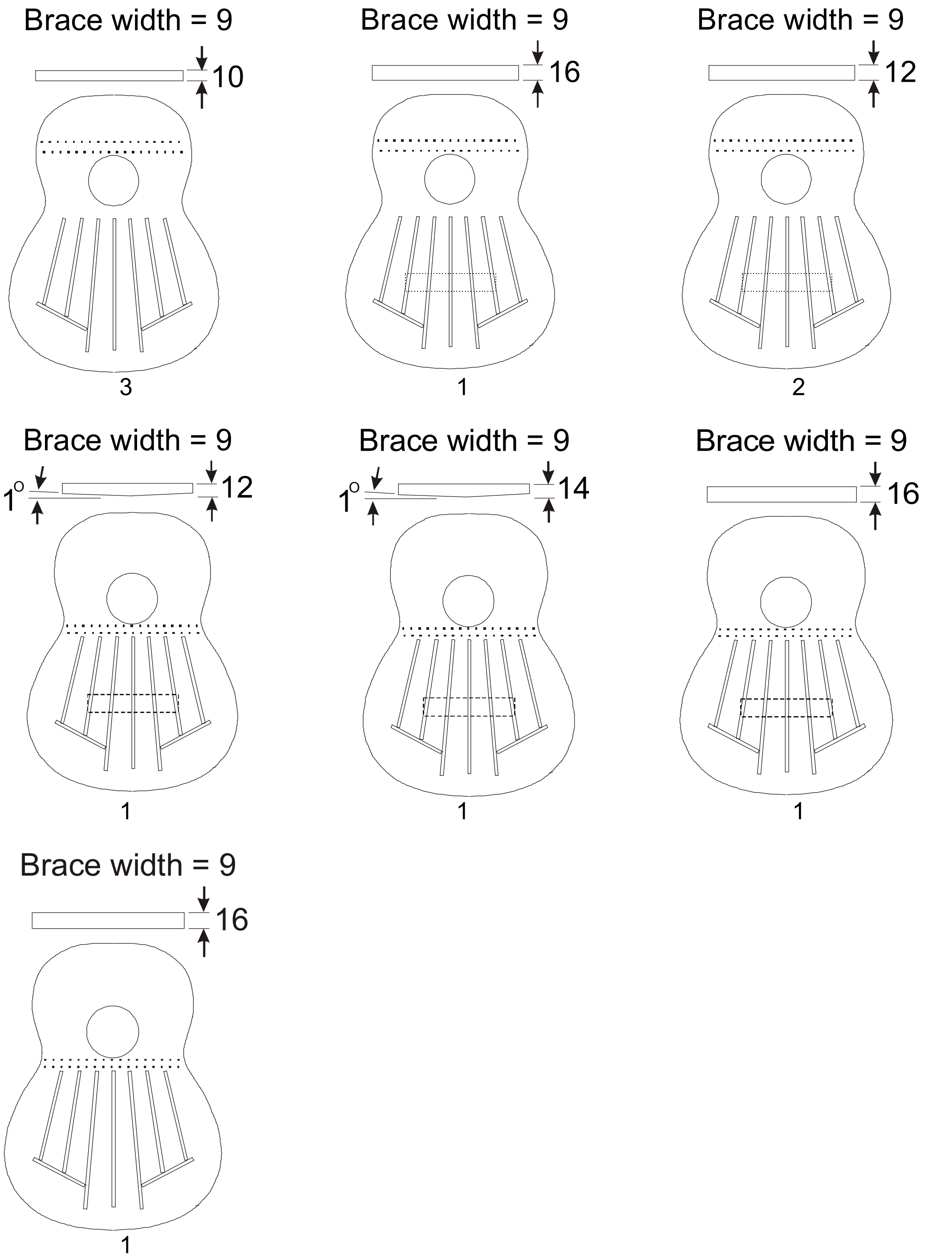


Figure 7b - Tested guitar tops, brace positions (doted lines) and tested braces. Number below each configuration indicates quantity of tested tops and braces. All dimensions are in millimeters. If glued on the top board, the bridge is indicated by a dashed line.

In spite of the fact that we have not a thorough physical explanation of the correlations between the measurements with the weight and brace gluing the following should agree with logic. As well known, modal mass should be rather lower than higher for all guitar modes [18]. In other words, light soundboards are preferred which is supported by a fact that a sort of inverse proportionality between their weight and sound quality is reported [19]. Next, without any doubt we can claim that a 20 gram weight on a top means an increase in its mass (non-desired) and brace means mostly added stiffness. This mass and stiffness should not be mixed with modal mass and stiffness. As already defined, the brace position which results in relatively low ratio $b'/m'\cdot k'$ and consequently in relatively high both amplitude and quality factor of p' can be seen as the aim of brace positioning. It is clear that for a finished guitar Q factor of any mode should not be too high [6], therefore the presented method should be applied only when Q factor of the first guitar mode has to be increased. It sounds reasonable therefore, that position where the added 20-gram weight (mass) results in relatively high $b(x,y)/m(x,y)\cdot k(x,y)$ for the analyzed mode, is most appropriate for gluing of brace (mostly stiffness) which results in relatively low $b'/m'\cdot k'$ for this mode. Maybe a satisfactory physical explanation of this phenomenon should be done well in combination with so called perturbation theory and consequently with perturbation function [20].

It has to be emphasized that the presented method does not enable a prediction of an absolute effect of the brace on the characteristics of the first peak in the FRF of a guitar. In other words, we cannot predict individual alterations in modal mass, stiffness and coefficient of viscous damping of system (*m-b-k-A*) after the brace gluing. For instance, too massive brace on positions with a lot of motion would increase modal mass drastically [12] which would most probably lead into a decrease of mode intensity. Therefore, the amplitude and quality factor of the first peak in the FRF of a guitar can also be decreased after the brace gluing, as indicated in Table 1. However, this decrease will always be smaller for a brace position defined by co-ordinates with relatively large ratios $b(x,y)/m(x,y)\cdot k(x,y)$. Almost all of the tested braces had a constant cross section. This explains the eventual decrease in amplitude and Q factor after the brace gluing which would most probably not occur in case of diminished brace height at its ends. Finally, no correlations between parameters of system (*m-b-k-A*) and frequency of p' after the brace gluing were estimated.

	1) Before brace gluing						*2) After brace gluing*			
	Without weight				With weight					
	$\frac{b'}{m'\cdot k'}$	P'	d'	f'_{0d}	Brace position r	$\frac{b(r)}{m(r)\cdot k(r)}$	$\frac{b'}{m'\cdot k'}$	P'	d'	f'_{0d}
	s/kg	Pa		Hz		s/kg	s/kg	Pa		Hz
1	1.31e-4	0.072	0.0216	85.3	r1	1.04e-4	1.50e-4	0.067	0.024	84.3
					r2	1.94e-4	1.44e-4	0.082	0.021	84.7
2	1.94e-4	0.079	0.0268	93.5	r1	1.21e-4	2.70e-4	0.077	0.032	93.5
					r2	1.77e-4	2.33e-4	0.080	0.029	93.6
3	2.67e-4	0.104	0.0261	88.4	r1	2.47e-4	2.14e-4	0.130	0.021	93.2
					r2	2.38e-4	2.20e-4	0.125	0.022	93.0
4	3.16e-4	0.133	0.025	90.2	r1	2.71e-4	2.37e-4	0.157	0.021	94.4
					r2	2.98e-4	1.85e-4	0.168	0.018	93.5
5	2.40e-4	0.100	0.026	94.1	r1	1.31e-4	1.14e-4	0.073	0.024	116.3
					r2	1.58e-4	1.07e-4	0.075	0.023	117.0
6	2.42e-4	0.121	0.025	96.9	r1	1.58e-4	1.25e-4	0.077	0.024	115.9
					r2	2.30e-4	0.98e-4	0.081	0.021	116.5
7	1.11e-4	0.068	0.024	115.9	r1	0.95e-4	1.08e-4	0.083	0.022	116.3
					r2	1.32e-4	1.03e-4	0.088	0.020	115.6
8	1.14e-4	0.044	0.025	84.3	r1	1.31e-4	0.97e-4	0.058	0.022	93.8
					r2	1.16e-4	1.08e-4	0.047	0.026	94.7
9	0.78e-4	0.044	0.026	115.3	r1	1.46e-4	0,76e-4	0.045	0.025	115.4
					r2	0.82e-4	1.60e-4	0.031	0.043	113.4
10	1.94e-4	0.184	0.018	94.9	r1	1.44e-4	1.34e-4	0.113	0.020	113.9
					r2	1.16e-4	1.60e-4	0.105	0.023	114.5

Table 1 - Correlations between experimentation with the weight and brace gluing.

IV. Conclusion

An analysis of the FRF, which is a ratio of sound pressure at 1 m from a guitar to the impulsive excitation of a guitar at the bridge, is based on the first resonant peak. This peak corresponds to a normal mode whose energy is radiated through the soundhole and is a result of interaction of both top and back guitar boards and the air between these boards. The analyzed peak which in fact indicates the ratio of the output to input signal was considered as a Fast Fourier Transformation of *p*'(*t*). The latter denotes sound pressure alteration in dependence on time, which is proportional to the damped oscillation of a virtual and hybrid mechanical-acoustic system (*m*-*b*-*k*-*A*). This consists of a mass, damper, spring, and surface with an area equal to the area of a soundhole (surface *A*). From the amplitude, frequency and damping of *p*', a discrete mass (*m*), coefficient of viscous damping (*b*) and stiffness (*k*) were calculated with high accuracy.

In dependence on its position, a 20-gram weight on the board differently causes the changes in *m*, *b* and *k* when these are compared to the situation without the weight on the board. Experiments showed that for the brace on a position defined by the places (*x*,*y*) with a relatively high ratio $b(x,y)/m(x,y)\cdot k(x,y)$, *p*' is more favorable in comparison to the brace position defined by places where this ratio is relatively low. By more favorable we have in mind peak amplitude and damping, and consequently loudness of guitar tones, at least those ones with frequencies close to the frequency of *p*' [5]. In addition, a relatively high ratio $b(x,y)/m(x,y)\cdot k(x,y)$ measured during experimentation with the weight always resulted in a relatively high ratio of amplitude to damping of *p*' after the brace gluing [2]. Because a decrease in ratio $b'/m'\cdot k'$ was our aim, and because light soundboards are preferred, it sounds logical that positions where the added mass has a non-desired effect on this ratio are most appropriate for the brace (mostly added stiffness). No correlations between parameters of system (*m*-*b*-*k*-*A*) and frequency of *p*' after the brace gluing were estimated.

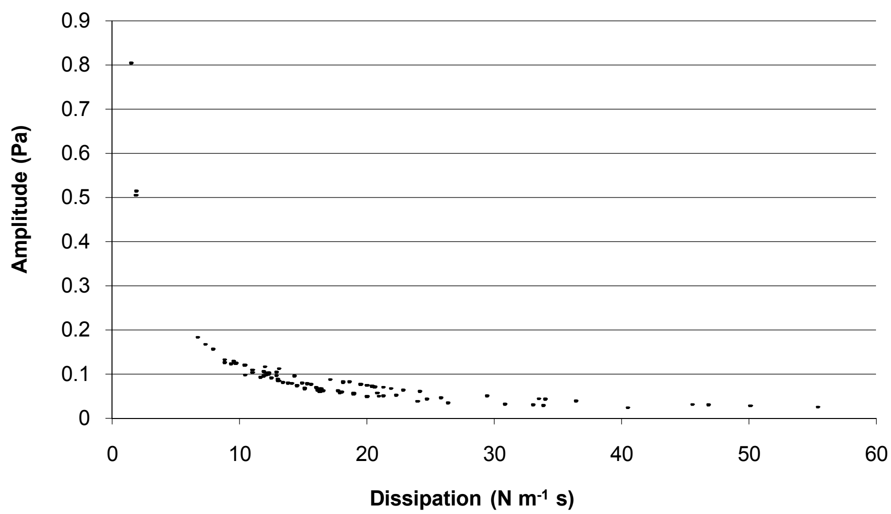


Figure 8 - Amplitude of the first peak in the FRF of a guitar versus dissipation (impedance), presented by coefficient of viscous damping.

A strong correlation between the amplitude of *p*' and coefficient of viscous damping *b* of system (*m*-*b*-*k*-*A*) was measured which strongly supports a physical adequacy of the presented method (see Figure 8). On the other side, the presented method does not enable a prediction of an absolute effect of the brace on *p*'. The reason for this lies in unknown individual alterations in modal mass, stiffness and coefficient of viscous damping of system (*m*-*b*-*k*-*A*) after the brace gluing. Thus, for the improper shape of a brace the amplitude of the first peak in the FRF of a guitar can also decreases after the brace gluing. However this decrease will always be smaller for a brace position defined by co-ordinates with relatively large ratios $b(x,y)/m(x,y)\cdot k(x,y)$. Despite of a lack of satisfactory physical explanation of the presented correlations between the effect of the weight and that of the brace, we can conclude that this model enables measuring of mechanical (acoustic) impedance of the first guitar mode, see Figure 8. A similar approach for other modes would most probably lead into a thorough definition of impedance characteristics of a guitar and consequently into a method for a complete brace optimization (not only in terms of tone loudness).

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